Konstruování a výpočty kompozitních konstrukcí

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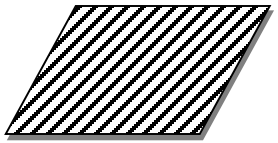
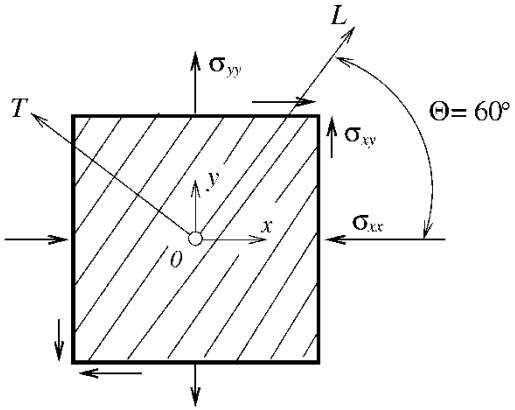
Obsah

- Laminátové desky
- Tyčové profily – pruty
- Sendviče
- Pevnost a únava vláknových kompozitů

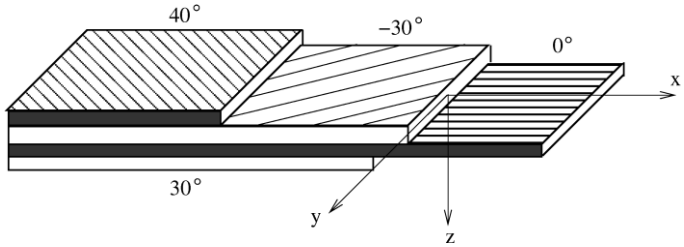
Tenké laminátové desky

Lamina



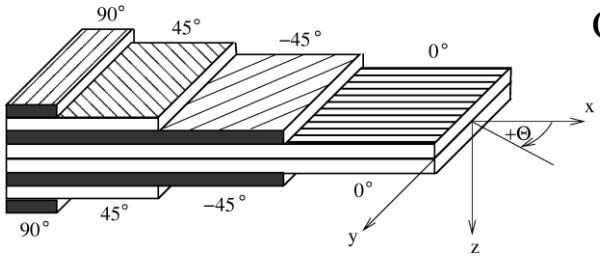


Laminát



Označení

$[40\,|-30|\,0\,|30]$



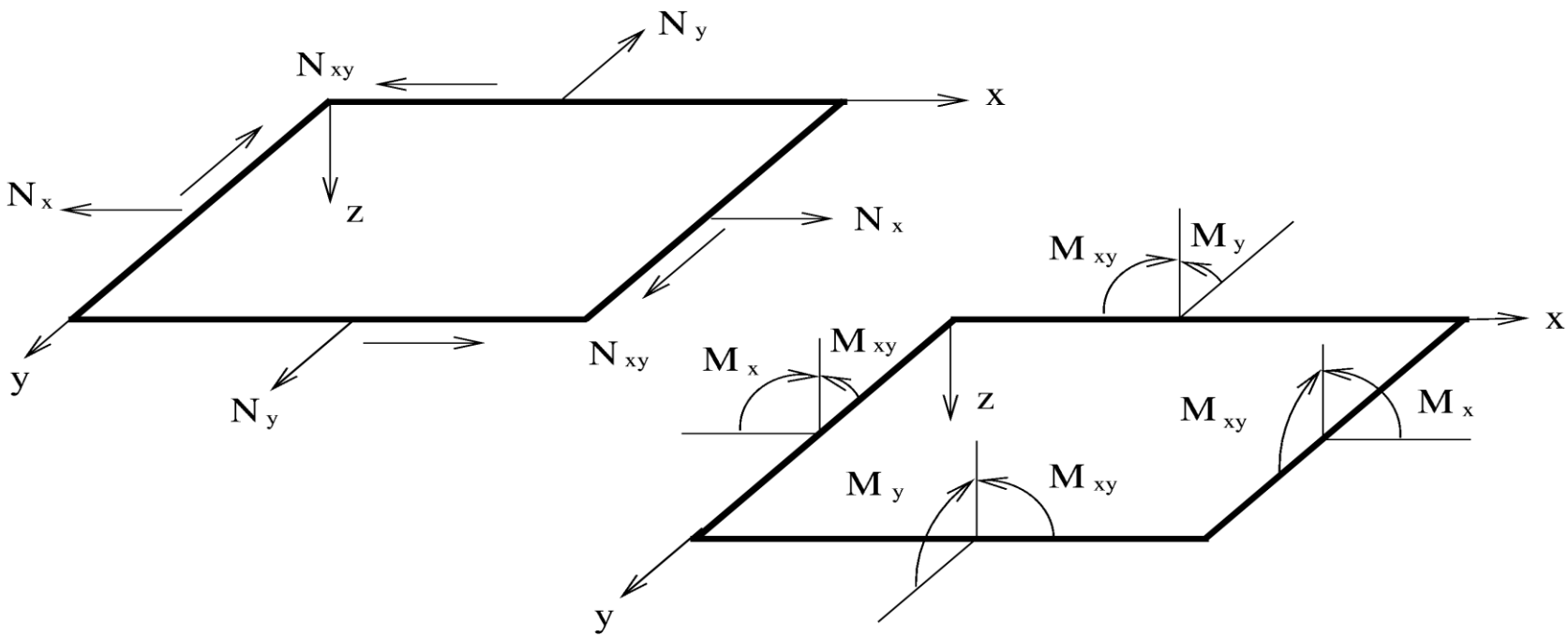
Označení

$[90\,|\pm 45|\,0]_S$

Tenké laminátové desky

Silové a momentové účinky na jednotku délky desky

N_{ij} [N/m], M_{ij} [Nm/m=N]



Klasická lamináční teorie (KLT)

Předpoklady řešení

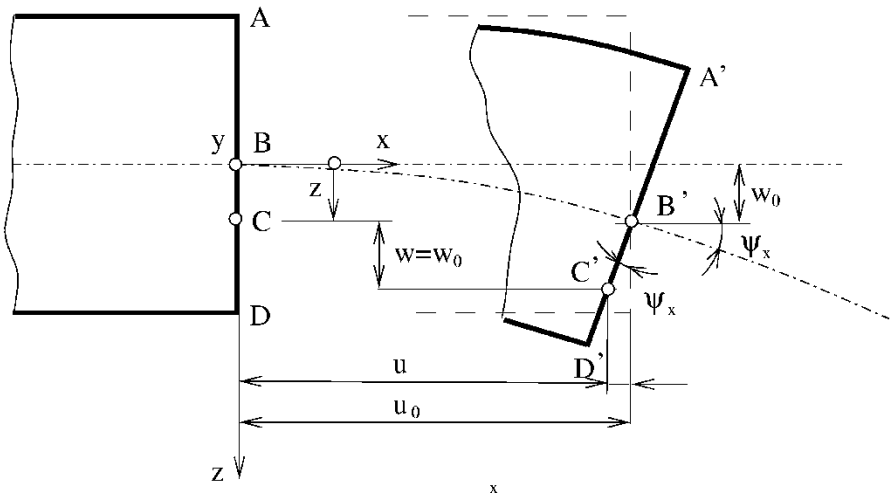
- každá lamina je ortotropní a kvazihomogenní
- tloušťka laminy je ve srovnání s délkou a šířkou velmi malá
- posunutí jednotlivých bodů ve všech třech směrech jsou malá
- spoj mezi laminami je dokonalý, nekonečně tenký a proto jsou posunutí spojitá
- posunutí se v příčném směru (po tloušťce) mění lineárně
- vzhledem k tomu, že tloušťka laminátu je vzhledem k ostatním rozměrům malá, je možno uvažovat rovinný stav napjatosti
- příčná zkosení $\gamma_{xz} = \gamma_{yz} \doteq 0$, a proto zůstanou kolmice ke středové ploše kolmé i po deformaci a budou přímkové
- normálová vzdálenost od středové roviny zůstává konstantní, a proto lze zanedbat přetvoření v příčném směru $\varepsilon_{zz} \doteq 0$
- závislosti mezi deformacemi a napětími budou lineární

Klasická lamináční teorie (KLT)

$$u = u_0 - z\psi_x \qquad \psi_x = \frac{\partial w_0}{\partial x}$$

$$u = u_0 - z\,\frac{\partial w_0}{\partial x}$$

$$v = v_0 - z\,\frac{\partial w_0}{\partial y}$$



$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\,\frac{\partial^2 w_0}{\partial x^2}$$

deformace na střednicové rovině

křivost

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} - z\,\frac{\partial^2 w_0}{\partial y^2}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\,\frac{\partial^2 w_0}{\partial x\,\partial y}$$

KLT – napětí v lamině

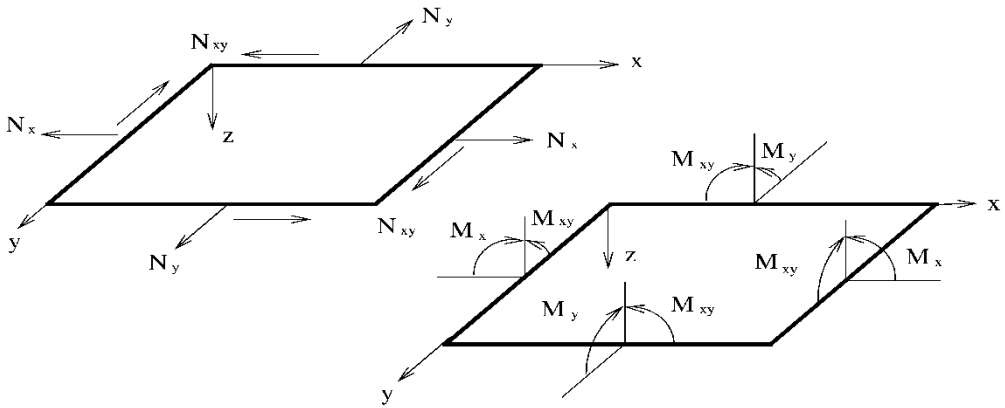
$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \end{bmatrix} + z \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

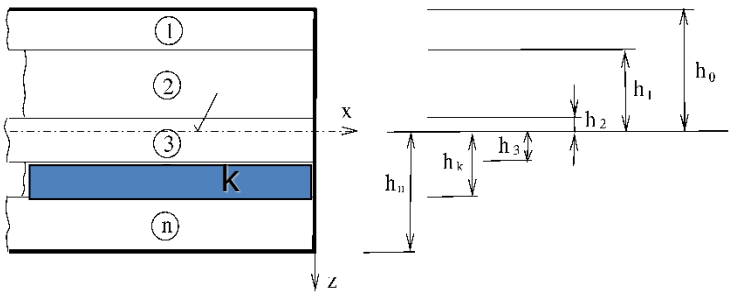
$$\boldsymbol{\varepsilon}^{\circ}_m = \begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{bmatrix} \qquad \mathbf{k} = \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} = -\begin{bmatrix} \dfrac{\partial^2 w_0}{\partial x^2} \\ \dfrac{\partial^2 w_0}{\partial y^2} \\ 2\dfrac{\partial^2 w_0}{\partial x\, \partial y} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix}_k = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \end{bmatrix} + z \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

kde Q_{ij} jsou prvky matice mimoosové tuhosti k- té laminy

KLT – síly a momenty v lamině





$$N_x = \int_{h_{k-1}}^{h_k} \sigma_{xx}\, dz, \quad N_y = \int_{h_{k-1}}^{h_k} \sigma_{yy}\, dz, \quad N_{xy} = \int_{h_{k-1}}^{h_k} \sigma_{xy}\, dz$$

$$M_x = \int_{h_{k-1}}^{h_k} \sigma_{xx}\, z\, dz, \quad M_y = \int_{h_{k-1}}^{h_k} \sigma_{yy}\, z\, dz, \quad M_{xy} = \int_{h_{k-1}}^{h_k} \sigma_{xy}\, z\, dz$$

KLT – síly a momenty v lamině

$$\mathbf{N} = \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} dz$$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix}_k = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \end{bmatrix} + z \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \sum_{k=1}^{n} \left\{ \int_{h_{k-1}}^{h_k} \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \end{bmatrix} dz + \int_{h_{k-1}}^{h_k} \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} z \, dz \right\}$$

KLT – síly a momenty v lamině

$$\mathbf{M} = \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \sum_{k=1} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} z \; dz$$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix}_k = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \end{bmatrix} + z \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \sum_{k=1}^{n} \left\{ \int_{h_{k-1}}^{h_k} \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \end{bmatrix} z \; dz + \int_{h_{k-1}}^{h_k} \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} z^2 \; dz \right\}$$

KLT – matice tuhosti laminátu

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \\ k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

$$A_{ij} = \sum_{k=1}^{n} \left(Q_{ij}\right)_k \left(h_k - h_{k-1}\right)$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \left(Q_{ij}\right)_k \left(h_k^2 - h_{k-1}^2\right)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \left(Q_{ij}\right)_k \left(h_k^3 - h_{k-1}^3\right)$$

$$\begin{bmatrix} \mathbf{N} \\ \cdots \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \vdots & \mathbf{B} \\ \cdots & \vdots & \cdots \\ \mathbf{B} & \vdots & \mathbf{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}^{\circ}_{\mathbf{m}} \\ \cdots \\ \mathbf{k} \end{bmatrix}$$

KLT – určení deformací ze zatížení

Inverzní relace – určení deformací (relat. prodloužení a křivosti střednicové roviny)

$$\begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \\ k_x \\ k_y \\ k_{xy} \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} & \bar{A}_{16} & \bar{B}_{11} & \bar{B}_{12} & \bar{B}_{16} \\ \bar{A}_{21} & \bar{A}_{22} & \bar{A}_{26} & \bar{B}_{21} & \bar{B}_{22} & \bar{B}_{26} \\ \bar{A}_{61} & \bar{A}_{62} & \bar{A}_{66} & \bar{B}_{61} & \bar{B}_{62} & \bar{B}_{66} \\ \bar{B}_{11} & \bar{B}_{12} & \bar{B}_{16} & \bar{D}_{11} & \bar{D}_{12} & \bar{D}_{16} \\ \bar{B}_{21} & \bar{B}_{22} & \bar{B}_{26} & \bar{D}_{21} & \bar{D}_{22} & \bar{D}_{26} \\ \bar{B}_{61} & \bar{B}_{62} & \bar{B}_{66} & \bar{D}_{61} & \bar{D}_{62} & \bar{D}_{66} \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{\varepsilon}^{\circ}_{\mathbf{m}} \\ \cdots \\ \mathbf{k} \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{A}} & \vdots & \bar{\mathbf{B}} \\ \cdots & \vdots & \cdots \\ \bar{\mathbf{B}} & \vdots & \bar{\mathbf{D}} \end{bmatrix} \begin{bmatrix} \mathbf{N} \\ \cdots \\ \mathbf{M} \end{bmatrix}$$

KLT – relace deformace a zatížení

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \left[\sum_{k=1}^{n} \mathbf{Q}_k \int_{h_{k=1}}^{h_k} dz \right] \begin{bmatrix} \varepsilon_{xx}^{\circ} \\ \varepsilon_{yy}^{\circ} \\ \gamma_{xy}^{\circ} \end{bmatrix} + \left[\sum_{k=1}^{n} \mathbf{Q}_k \int_{h_{k=1}}^{h_k} z \, dz \right] \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \left[\sum_{k=1}^{n} \mathbf{Q}_k \int_{h_{k=1}}^{h_k} z \, dz \right] \begin{bmatrix} \varepsilon_{xx}^{\circ} \\ \varepsilon_{yy}^{\circ} \\ \gamma_{xy}^{\circ} \end{bmatrix} + \left[\sum_{k=1}^{n} \mathbf{Q}_k \int_{h_{k=1}}^{h_k} z^2 \, dz \right] \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

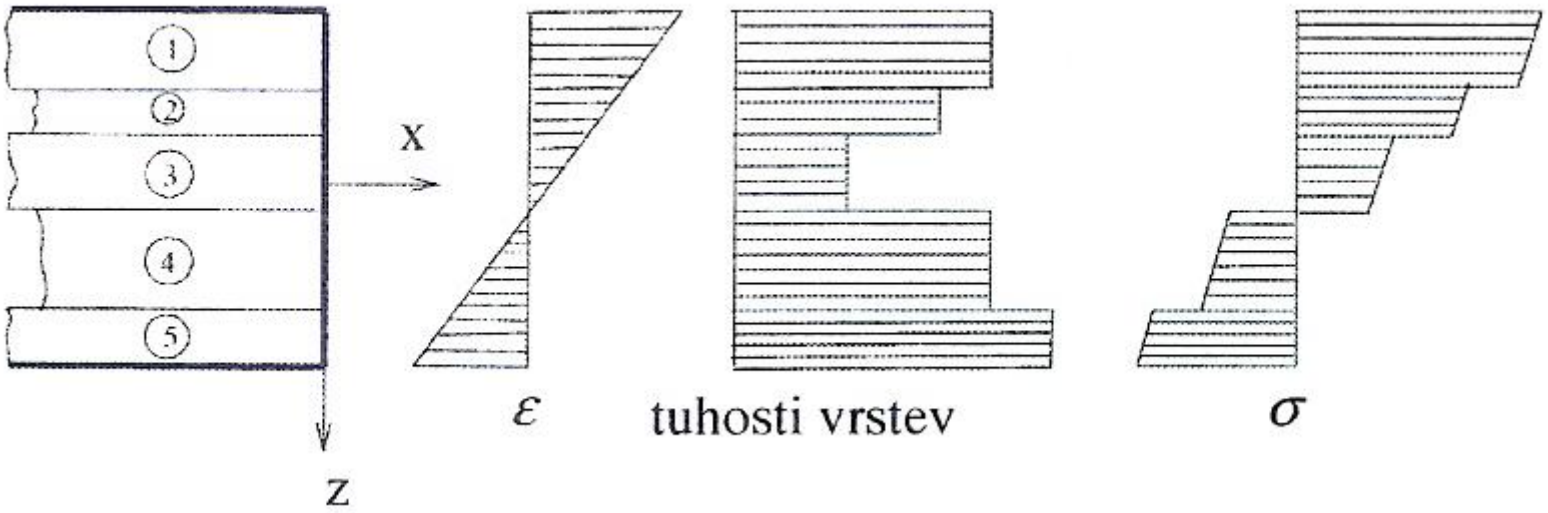
$$A_{ij} = \sum_{k=1}^{n} \left(Q_{ij} \right)_k \left(h_k - h_{k-1} \right)$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \left(Q_{ij} \right)_k \left(h_k^2 - h_{k-1}^2 \right)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \left(Q_{ij} \right)_k \left(h_k^3 - h_{k-1}^3 \right)$$

Průběhy napětí a deformací

Příklad průběhů deformace, tuhosti a napětí v laminátu

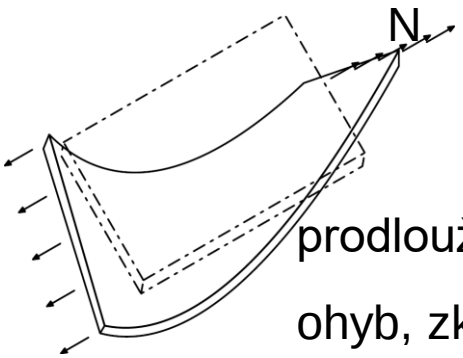


KLT – příklady

Obecné lamináty

(zatížením vzniknou kombinované módy prodloužení a křivostí)

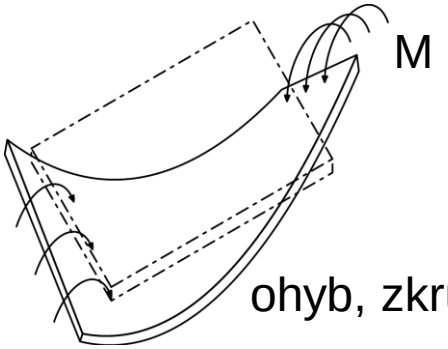
$$Např.: \quad \begin{bmatrix} 40 & |-30| & 0 & |60| \end{bmatrix}$$



prodloužení, zkos,

ohyb, zkrut

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \\ k_x \\ k_y \\ k_{xy} \end{bmatrix}$$



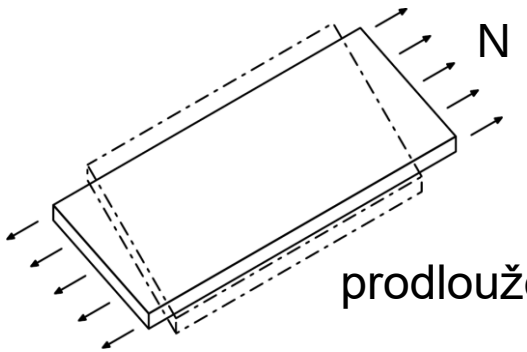
ohyb, zkrut,

prodloužení, zkos

KLT – příklady

Symetrické lamináty (odstraňují vazbu mezi silovými a momentovými účinky)

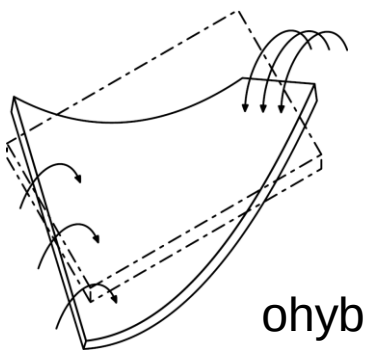
$$Např.: \quad \left[45 \quad |90| \quad 0 \quad |60| -30\right]_S \quad s\ různou\ tloušťkou$$



prodloužení, zkos

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{n}\left(Q_{ij}\right)_k\left(h_k^2 - h_{k-1}^2\right) = 0$$

$$\begin{bmatrix} A_{11} & A_{12} & A_{16} & 0 & 0 & 0 \\ A_{21} & A_{22} & A_{26} & 0 & 0 & 0 \\ A_{61} & A_{62} & A_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{11} & D_{12} & D_{16} \\ 0 & 0 & 0 & D_{21} & D_{22} & D_{26} \\ 0 & 0 & 0 & D_{61} & D_{62} & D_{66} \end{bmatrix}$$

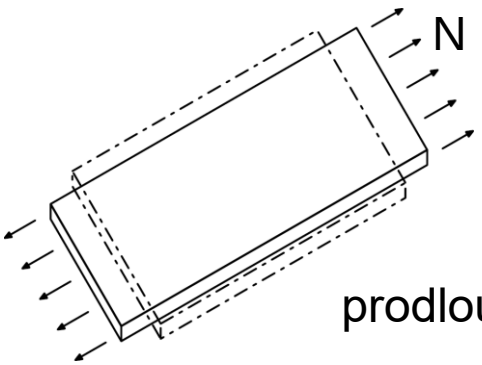


ohyb, zkrut

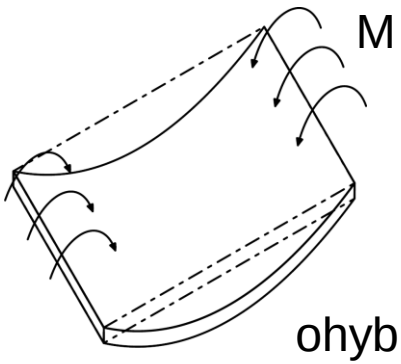
$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{21} & D_{22} & D_{26} \\ D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

KLT – příklady

Symetrické křížově vrstvené lamináty (mají vrstvy 0º a 90º symetricky)
(není vazba mezi silami a momenty ani mezi normálovými a smykovými účinky)



prodloužení



ohyb

$$Např.: \quad \left[0 \quad |90| \quad 0 \quad |90|\right]_S \quad s\ různou\ tloušťkou$$

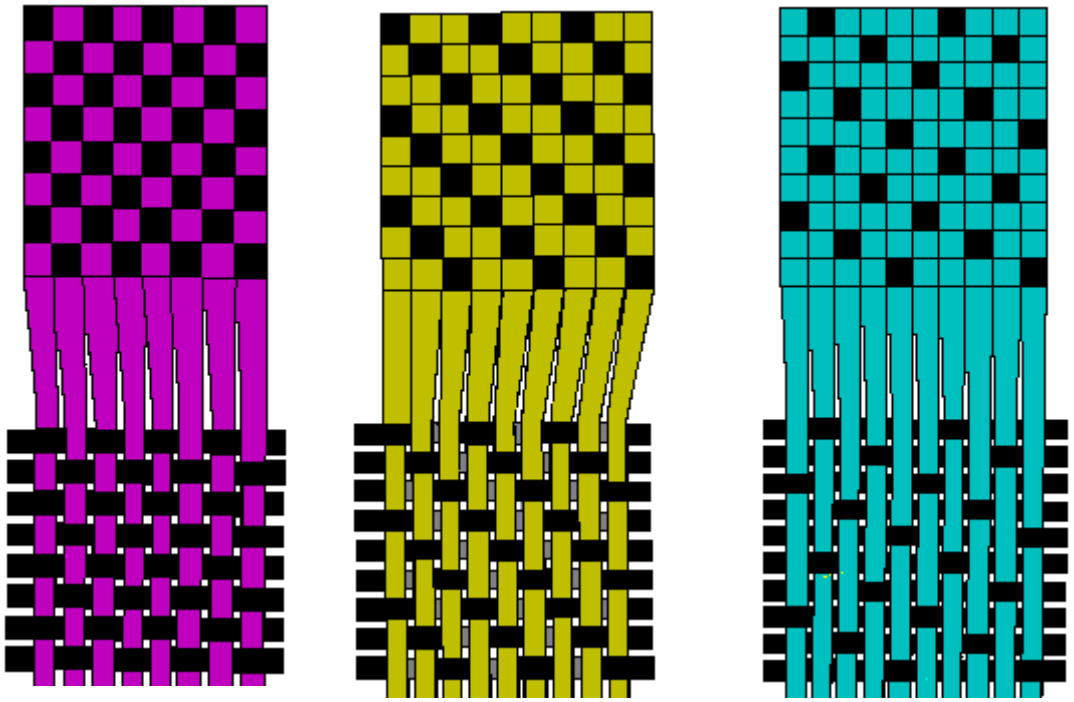
$$B_{ij} = 0 \ , \ A_{16} = A_{26} = 0, \ D_{16} = D_{26} = 0 \ , \ i = 1, \ 2, \ 6$$

$$\begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ A_{21} & A_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{11} & D_{12} & 0 \\ 0 & 0 & 0 & D_{21} & D_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{66} \end{bmatrix}$$

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \end{bmatrix} \qquad \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

Tkaniny – typy vazeb

- **Základní vazby**
- plátnová, keprová a saténová

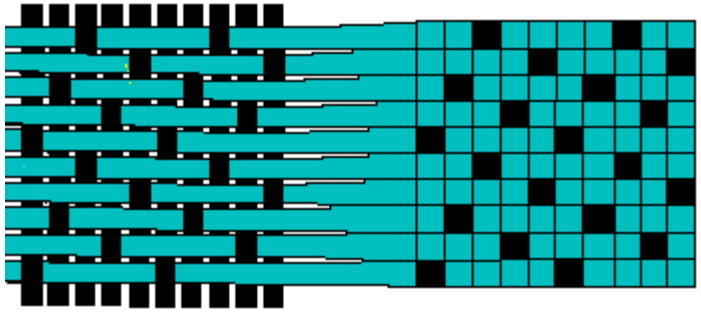


Tkaniny – ekvivalentní tloušťky

- **Určení ekvivalentní tloušťky jednosměrné vrstvy**

$$t_{warp} = t \cdot \frac{n_1}{n_1 + n_2} = k \cdot t$$

$$t_{fill} = t \cdot \frac{n_2}{n_1 + n_2} = (1 - k) \cdot t$$



- kde
- t celková tloušťka tkaniny
- n_1 počet pramenců osnovy (wa
- n_2 počet pramenců útku (fill)

Tkaniny – ekvivalentní elastické moduly

- **Určení ekvivalentních elastických modulů**

1. Pro objemový podíl vláken v tkanině V_f určíme vlastnosti laminy

$$E_L = V_f \cdot E_f + \left(1 - V_f\right) \cdot E_m$$

$$E_T = \frac{G_m}{1 - V_f \cdot \left(1 - \frac{G_m}{G_f}\right)}$$

$$G_{LT} = \frac{E_m}{1 - V_f \cdot \left(1 - \frac{E_m}{E_f}\right)}$$

$$\nu_L = V_f \cdot \nu_f + \left(1 - V_f\right) \cdot \nu_m$$

2. Tkaninu nahradíme jednou

- ekvivalentní vrstvou tl. t

Tkaniny – ekvivalentní elastické moduly

3. s modulem ve směru osnovy

$$E_x = k \cdot E_L + (1-k) \cdot E_T$$

4. ve směru útku

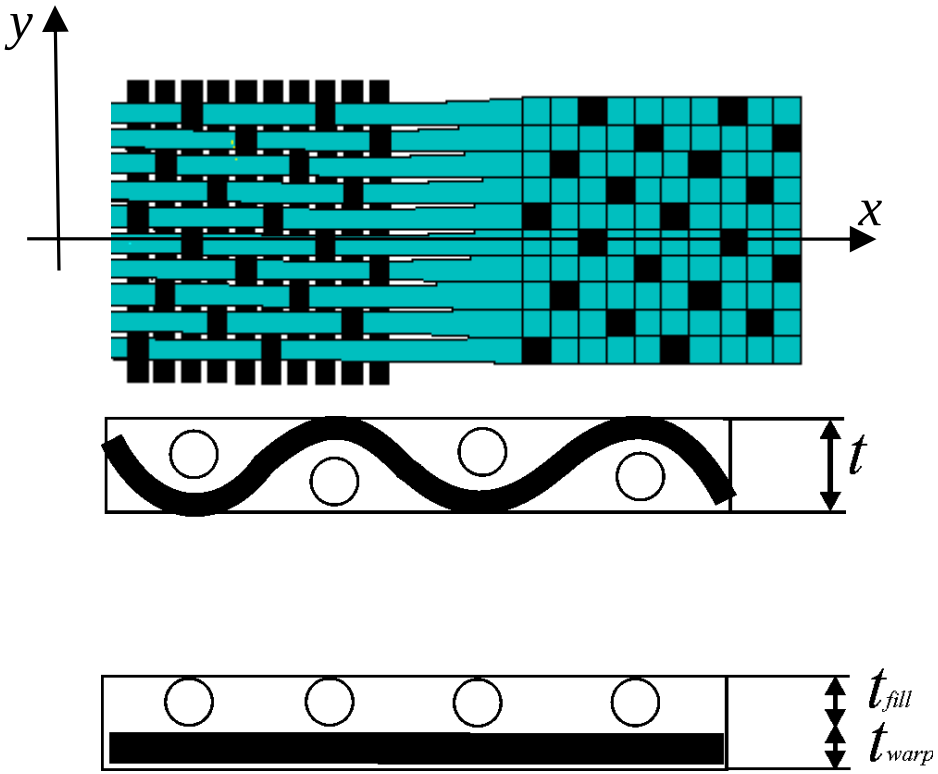
$$E_y = (1-k) \cdot E_L + k \cdot E_T$$

5. Se smykovým modulem

$$G_{xy} = G_{LT}$$

6. a s Poissonovým číslem

$$\nu_{xy} = \frac{\nu_{LT}}{k + (1-k) \cdot \left(\frac{E_L}{E_T} \right)}$$



Obsah

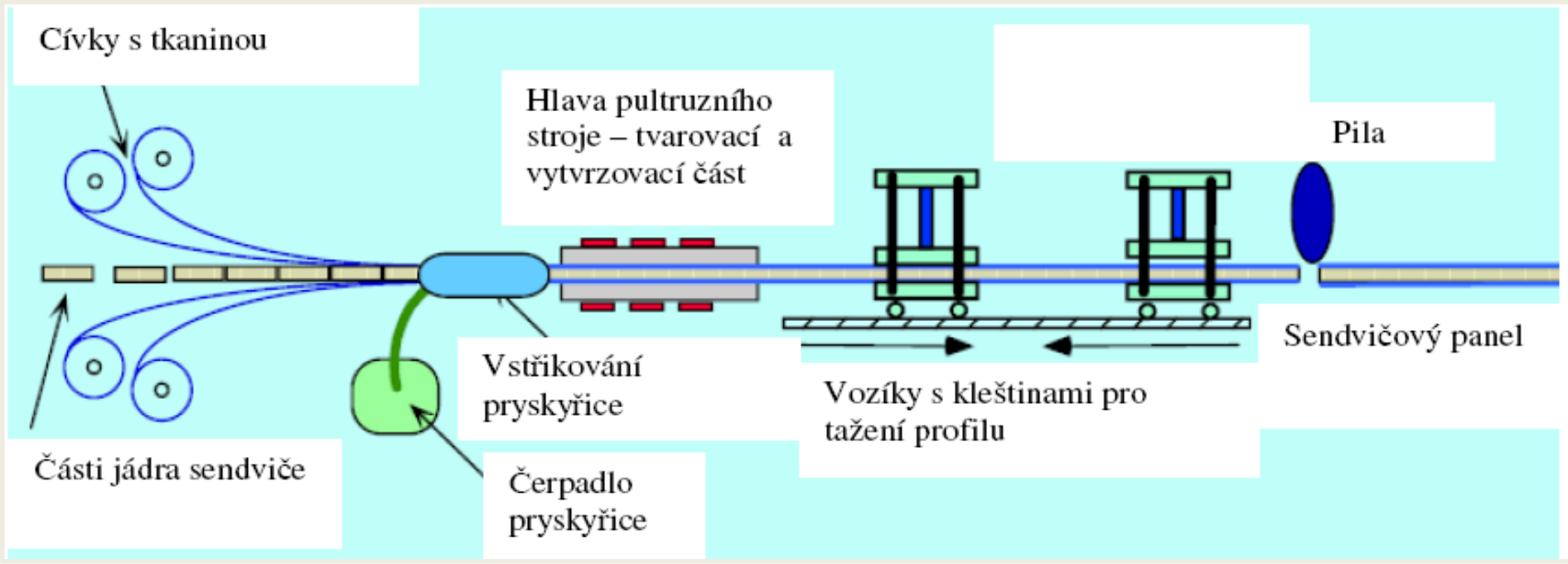
- Laminované desky
- Tyčové profily – pruty
- Sendviče
- Pevnost a únava vláknových kompozitů

Tyčové profily = pruty

Výroba:

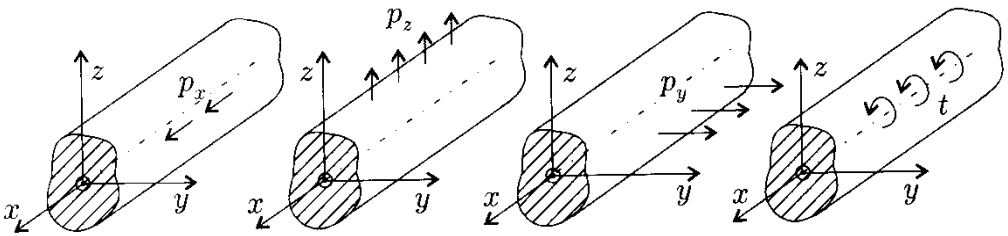
- ruční laminace (např. podélné kladení svazků vláken)
- pultruze
- navíjení (kruhové i nekruhové profily)
- tkaní (2D i 3D) struktury

Tyčové profily - výroba

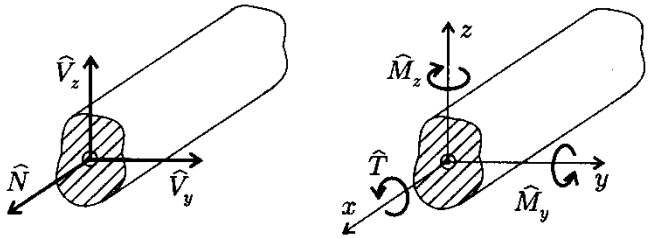


Pruty - označení

- Souřadnicový systém a zatížení:



Vnitřní silové účinky:

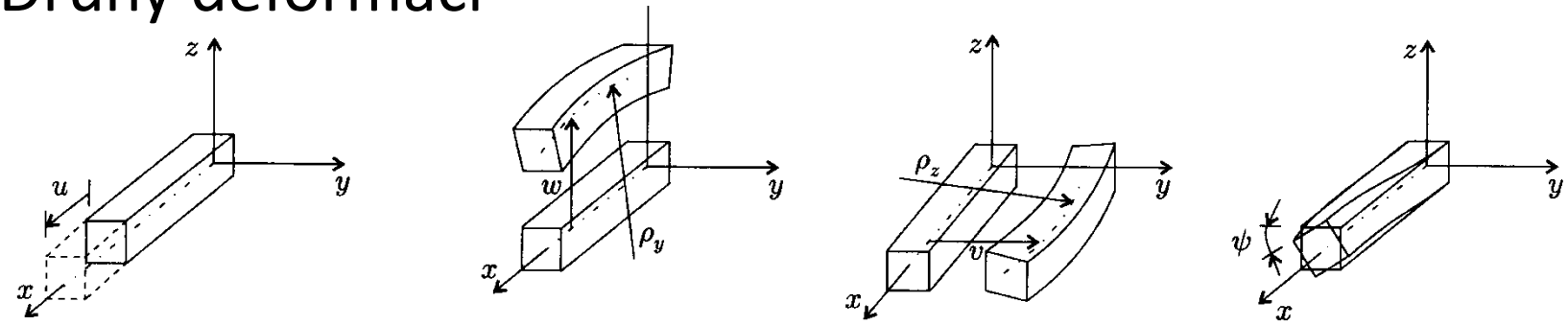


$$-p_x = \frac{\partial \hat{N}}{\partial x}, \quad -p_y = \frac{\partial \hat{V}_y}{\partial x}, \quad -p_z = \frac{\partial \hat{V}_z}{\partial x}$$

$$-t = \frac{\partial \hat{T}}{\partial x}, \quad \hat{V}_z = \frac{\partial \hat{M}_y}{\partial x}, \quad \hat{V}_y = \frac{\partial \hat{M}_z}{\partial x}$$

Pruty - deformace

Druhy deformací



$$\varepsilon_x^0 = \frac{\partial u}{\partial x}, \quad k_y = \frac{1}{\rho_y} = \frac{\partial^2 w}{\partial x^2}, \qquad k_z = \frac{1}{\rho_z} = \frac{\partial^2 v}{\partial x^2}, \quad \vartheta = \frac{\partial \psi}{\partial x}$$

prodloužení , křivost s poloměrem ρ_y , křivost s poloměrem ρ_z , zkrut

Pruty – matice tuhosti

Relace s maticí tuhosti

$$\begin{Bmatrix} \hat{N} \\ \hat{M}_y \\ \hat{M}_z \\ \hat{T} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{12} & D_{22} & D_{23} & D_{24} \\ D_{13} & D_{23} & D_{33} & D_{34} \\ D_{14} & D_{24} & D_{34} & D_{44} \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_x^0 \\ k_y \\ k_z \\ \vartheta \end{Bmatrix}$$

$$\begin{Bmatrix} \hat{N} \\ \hat{M}_y \\ \hat{M}_z \\ \hat{T} \end{Bmatrix} = \begin{bmatrix} EA & 0 & 0 & 0 \\ 0 & EJ_{yy} & EJ_{yz} & 0 \\ 0 & EJ_{yz} & EJ_{zz} & 0 \\ 0 & 0 & 0 & GJ_k \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_x^0 \\ k_y \\ k_z \\ \vartheta \end{Bmatrix}$$

$$\begin{Bmatrix} \hat{N} \\ \hat{M}_y \\ \hat{M}_z \\ \hat{T} \end{Bmatrix} = \begin{bmatrix} D_{11} & 0 & 0 & 0 \\ 0 & D_{22} & D_{23} & 0 \\ 0 & D_{23} & D_{33} & 0 \\ 0 & 0 & 0 & D_{44} \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_x^0 \\ k_y \\ k_z \\ \vartheta \end{Bmatrix}$$

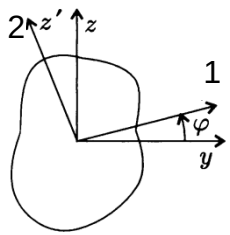
$$\begin{Bmatrix} \hat{N} \\ \hat{M}_y \end{Bmatrix} = \begin{bmatrix} \hat{EA} & 0 \\ 0 & \hat{EJ}_{yy} \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_x^0 \\ k_y \end{Bmatrix}$$

Pruty – ekvivalentní tuhost

$$Namísto \quad A, J_y, J_k \quad \rightarrow \quad ekvivalentní\ \hat{E}A,\ \hat{E}J_y\ , \hat{G}J_k$$

- Analogie v natočení os průřezu a v určování hlavních směrů

$$tg2\varphi = -\frac{2\hat{E}J_{yz}}{\hat{E}J_{yy} - \hat{E}J_{zz}}$$

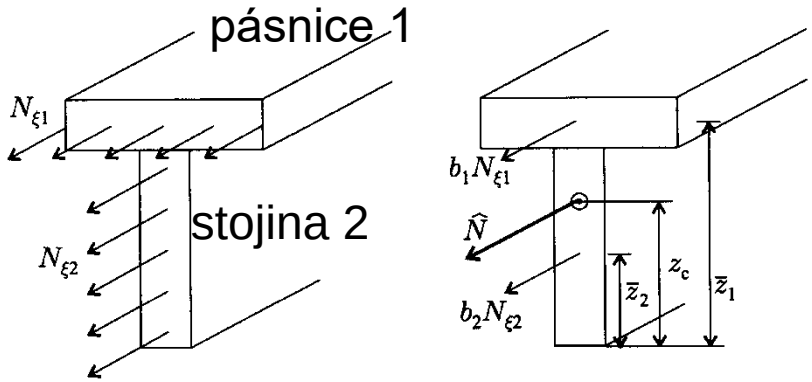


$$\hat{E}J_1 = \frac{\hat{E}J_{yy} + \hat{E}J_{zz}}{2} + \sqrt{\left(\frac{\hat{E}J_{yy} - \hat{E}J_{zz}}{2}\right)^2 + \hat{E}J_{yz}^{\ 2}}$$

Pruty - příklad

- Určení ekvivalentní tahové tuhosti

- Předpoklad $\varepsilon_{\xi 1} = \varepsilon_{\xi 1} = \varepsilon^{0}{}_{x}$



$$\hat{N} = b_1 N_{\xi 1} + b_2 N_{\xi 2}$$

$$\hat{N} = \underbrace{\left[\left(b_1 \cdot (EA)_1 + b_2 \cdot (EA)_2\right)\right]}_{\hat{EA}} \cdot \varepsilon_x^0$$

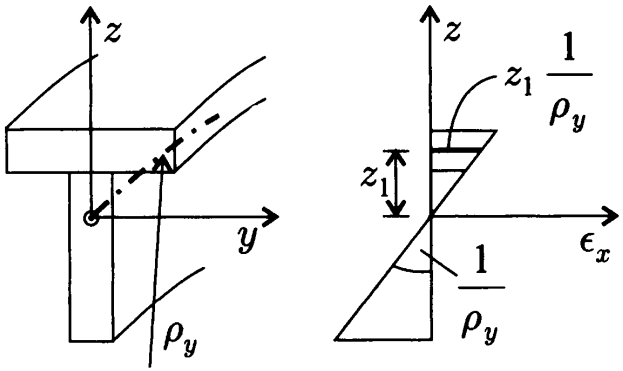
ekvivalentní $\hat{EA}$

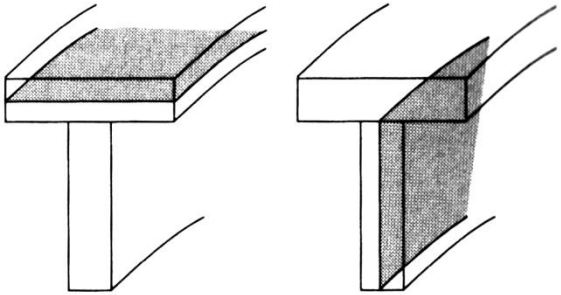
Pruty –příklad

- Deformace při ohybu

$$\varepsilon^0_{\xi 1} = \frac{1}{\rho_y} z_1 \qquad \varepsilon^0_{\xi 1} = \frac{1}{\rho_y} z$$

$$k^0_{\xi 1} = \frac{1}{\rho_y} \qquad k^0_{\xi 2} = 0$$



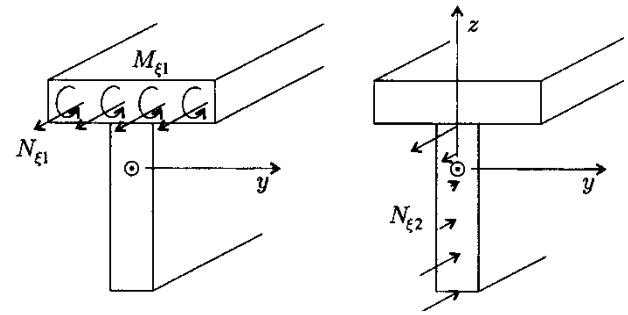


Pruty - příklad

- Určení ekvivalentní ohybové tuhosti

$$N_{\xi 1} = (EA)_1 \cdot \varepsilon_{\xi 1}^0 \qquad N_{\xi 2} = (EA)_2 \cdot \varepsilon_{\xi 2}^0$$

$$M_{\xi 1} = (EJ)_1 \cdot k_{\xi 1}$$



$$\hat{M} = b_1 N_{\xi 1} \cdot z_1 + b_1 M_{\xi 1} + \int_{b_2} N_{\xi 2} \cdot z \cdot dz$$

$$\hat{M} = \frac{1}{\rho_y} \underbrace{\left[b_1 (EA)_1 \cdot z_1^2 + b_1 (EJ)_1 + (EJ)_2 \left(\frac{b^3}{12} + z_2^2 \cdot b_2 \right) \right]}_{\text{ekvivalentní} \quad \hat{EJ}}$$

Obsah

- Laminované desky
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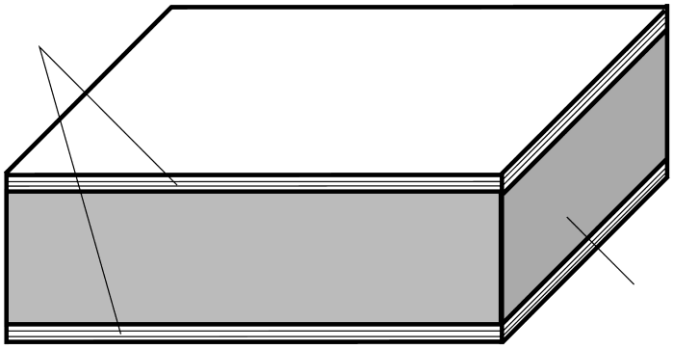
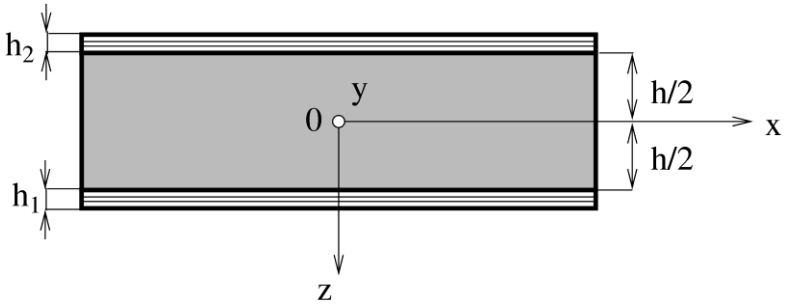
Sendviče

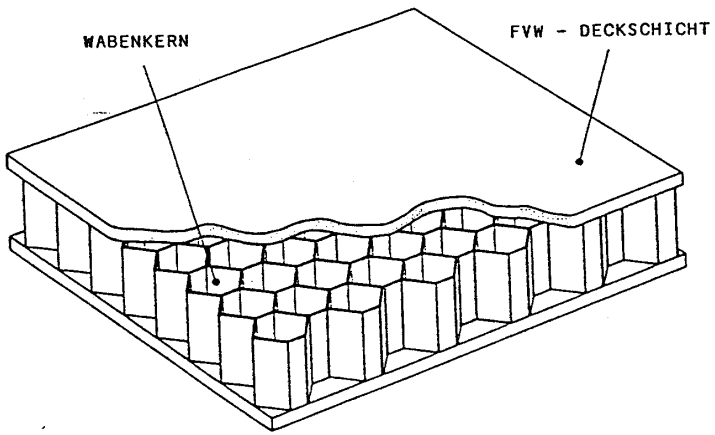
•Předpoklady řešení

- malé deformace a platnost Hookova zákona
- tloušťka jádra musí být mnohem větší než tloušťky potahů.
- posunutí jádra ve směru *x* a ve směru *y* se po tloušťce *z* mění lineárně
- posunutí potahů *u* a *v* jsou konstantní po celé jejich tloušťce
- příčné posunutí *w* je nezávislé na souřadnici *z* a proto je možno uvažovat
- jádro přenáší pouze příčná smyková napětí a tudíž
- vzhledem k tomu, že je tloušťka potahů malá, je možno na nich zanedbat příčná smyková napětí a normálové napětí ve směru *z,* čili.





Sendviče



Sendvič s jádrem Honeycomb

Materiály potahu Sklo/E, C/E laminát, Al-slit
Materiály jádra Pěny (PU, PVC),
Honeycomb (Aluminium, aramid paper (Nomex)
Dřevo (Balsa)

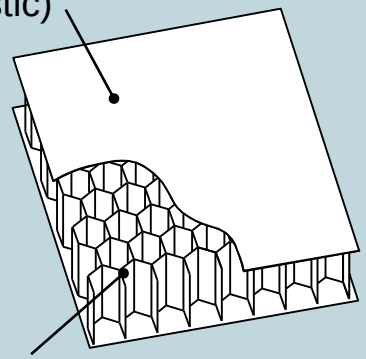
Aplikace:

- lehké konstrukce
- nosníky, desky, skořepiny
- konstrukce s vysokou tuhostí a nízkou hmotností
- konstrukce s tepelně-izolačními vlastnostmi a vysokým útlumem

Sendvičové konstrukce v letectví

Honeycomb sendvičové konstrukce

horní+dolní potah (fibre-reinforced plastic)



Honeycomb jádro

+ výhody

vysoká tuhost vůči hmotnosti

- nevýhody

impaktní zranitelnost

akumulace vlhkosti

EADS

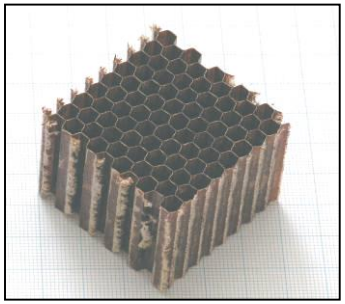
Photo: Airbus

Sendvičové konstrukce v letectví

Budoucí generace dopravních letadel s černým trupem (“black” CFRP fuselage)

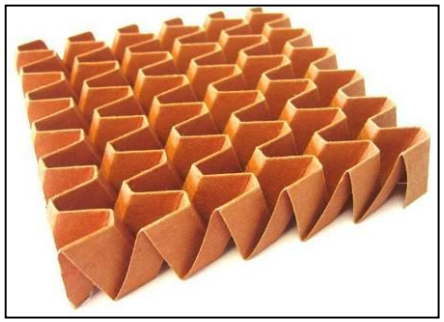
- 30 % úspora hmotnosti i nákladů
- Nevhodnost klasického řešení podélník-potah
- nová „dvoustěnná-sendvičová“ koncepce **double-walled fuselage structure =** C/E potahy, lehké voštinové jádro

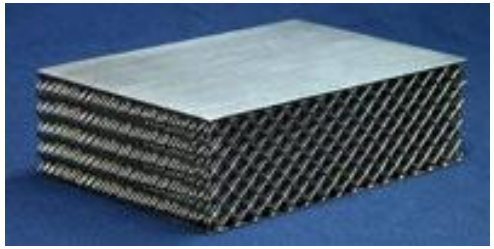
Nomex aramid paper honeycomb core



- Standardní sendvič používaný na sekundární díly
- Nízká impaktní energie
- Uzavřené buňky, vlhkost

Folded composite core





- Nové koncepce skládaných jader, kovové voštiny (i na primární díly)
- Otevřené buňky, vhodné pro trup

Návrh sendvičů

Charakteristiky pro navrhování sendviče na ohybové zatížení:

- Potah přenáší ohybové momenty, je tažen-tlačen, musí vykazovat vysokou tuhost a pevnost
- Jádro přenáší smyková zatížení a smykové napětí od potahu
- Jádro musí zajistit stabilitu potahů a zaručit integritu sendviče
- Tlusté jádro zvyšuje významně ohyb. tuhost růst kvadr. modulu Jy.
- Kvalitní spoj jádra a potahu musí zaručit dostatečnou odolnost vůči stabilitě potahu
- Load introduction requires joints which transfer loads to the skins
- Spoj s kovovými částmi musí zajistit přenos sil do potahů.

Deformace sendviče

Pole posuvů jádra je možno vyjádřit ve tvaru $u_c = u_0 - z\psi_x \; v_c = v_0 - z\psi_y$

Pole posuvů jádra lze rozdělit na dvě deformační pole

- deformační pole střední roviny a ohybové deformační pole

$$\begin{bmatrix} \varepsilon_{xx}^c \\ \varepsilon_{yy}^c \\ \gamma_{xy}^c \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx}^\circ \\ \varepsilon_{yy}^\circ \\ \gamma_{xy}^\circ \end{bmatrix} - z \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

- pole příčných smykových deformací

$$\boldsymbol{\gamma}^c = \begin{bmatrix} \gamma_{yz}^c \\ \gamma_{zx}^c \end{bmatrix} = \begin{bmatrix} \dfrac{\partial w_0}{\partial y} - \psi_y \\ \dfrac{\partial w_0}{\partial x} - \psi_x \end{bmatrix}$$

Napětí v sendviči

Je možno uvažovat **pole napětí jádra a potahů odděleně**

Protože napjatost jádra
působí na jádru pouze smyková napětí

$$\sigma_{xx}^{c} = \sigma_{yy}^{c} = \sigma_{xy}^{c} = \sigma_{zz}^{c} = 0$$

$$\begin{bmatrix} \sigma_{yz}^{c} \\ \sigma_{zx}^{c} \end{bmatrix} = \begin{bmatrix} C'^{c}_{44} & C'^{c}_{45} \\ C'^{c}_{54} & C'^{c}_{55} \end{bmatrix} \begin{bmatrix} \gamma_{yz}^{c} \\ \gamma_{zx}^{c} \end{bmatrix}$$

Pro potahy platí, že v k-té vrstvě:

$$\sigma_{zz}^{k} = \sigma_{zx}^{k} = \sigma_{yz}^{k} = 0$$

$$\begin{bmatrix} \sigma_{xx}^{k} \\ \sigma_{yy}^{k} \\ \sigma_{xy}^{k} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix}_{k} \begin{bmatrix} \varepsilon_{xx}^{l} \\ \varepsilon_{yy}^{l} \\ \varepsilon_{zz}^{l} \end{bmatrix}, \quad l = 1,\ 2$$

Silové účinky v sendviči

Výsledné síly působící na příčný řez sendviče:

$$\mathbf{N} = \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \int_{-\left(\frac{h}{2}+h_1\right)}^{-\frac{h}{2}} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} dz + \int_{\frac{h}{2}}^{\frac{h}{2}+h_2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} dz$$

$$\mathbf{M} = \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \int_{-\left(\frac{h}{2}+h_1\right)}^{-\frac{h}{2}} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} z\, dz + \int_{\frac{h}{2}}^{\frac{h}{2}+h_2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} z\, dz$$

Výslednice vnějších sil, působících na jádro kolmo na rovinu (x,y)

$$\mathbf{Q}_c = \begin{bmatrix} Q_y \\ Q_x \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_{zx}^c \\ \sigma_{yz}^c \end{bmatrix} dz$$

Silové účinky v sendviči

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_y \\ Q_x \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ C_{11} & C_{12} & C_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ C_{12} & C_{22} & C_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ C_{16} & C_{26} & C_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & F_{44} & F_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & F_{45} & F_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \\ k_x \\ k_y \\ k_{xy} \\ \gamma_{yz}^c \\ \gamma_{zx}^c \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{N} \\ \mathbf{M} \\ \mathbf{Q}_c \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{0} \\ \mathbf{C} & \mathbf{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{F} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}_m^0 \\ \mathbf{k} \\ \boldsymbol{\gamma}^c \end{bmatrix}$$

Obsah

- Laminované desky
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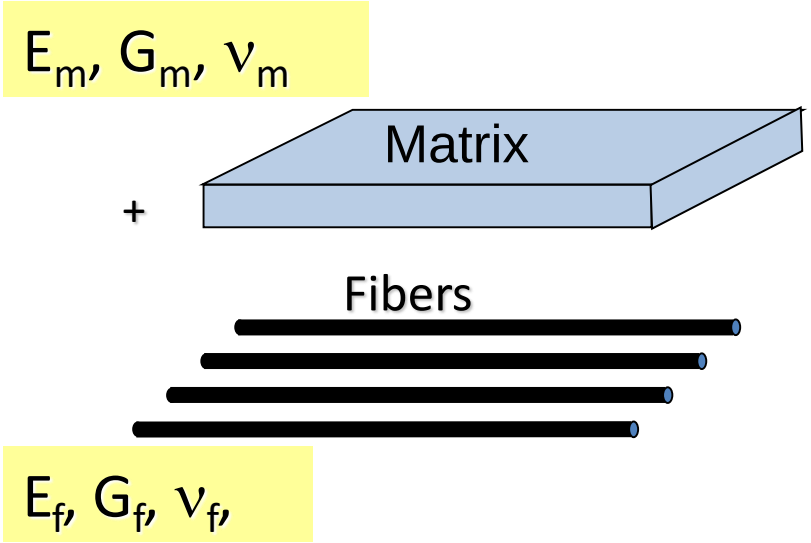
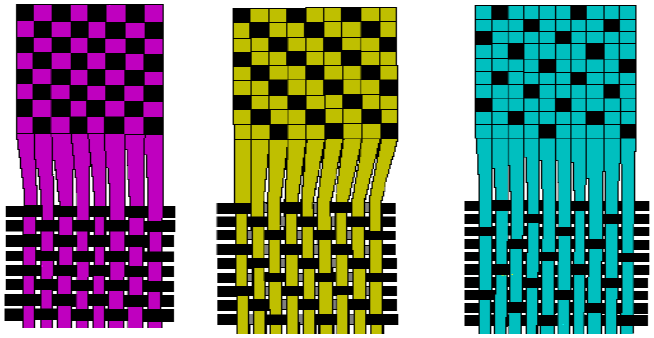
Composite laminates

We will focus on

- Polymer matrix composite
  - Fibers (carbon, glass, aramid ..) + epoxy matrix
- Laminated plates and shells
  - Long fiber unidirectional lamina (about 0.25 mm thin)
  - Laminate is formed by stacking of several laminae with different orientations
- Laminated plates and shells
  - Bi-directional woven or nonwoven fabric
  - Laminate is formed by stacking of several fabric with different orientations





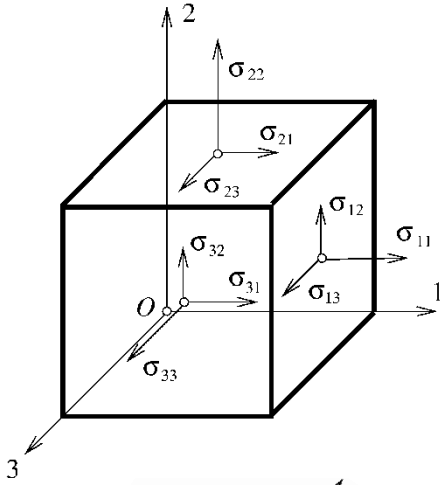
Properties of the materials

Table 1.1 Specific Modulus and Strength of Typical Fibers, Composites, and Bulk Metals

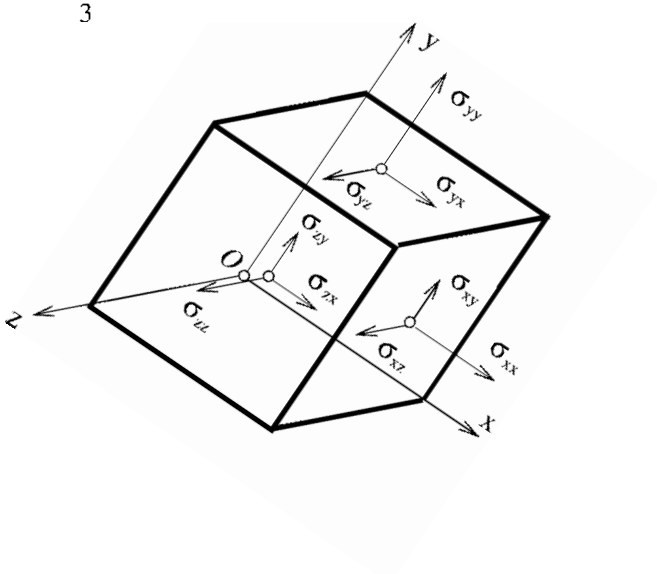
Material (Units)	Specific gravity	Young's modulus (GPa)	Ultimate strength (MPa)	Specific modulus (GPa-m³/kg)	Specific strength (MPa-m³/kg)
		System of Units: SI			
Graphite	1.8	230.00	2067	0.1278	1.148
Aramid	1.4	124.00	1379	0.08857	0.9850
Glass	2.5	85.00	1550	0.0340	0.6200

Constitutive characteristics

- Global and local coordinate systems Stress- strains relationships



$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix}$$



$$\boldsymbol{\sigma} = \mathbf{C} \quad \boldsymbol{\varepsilon}$$

C .... Stiffness matrix

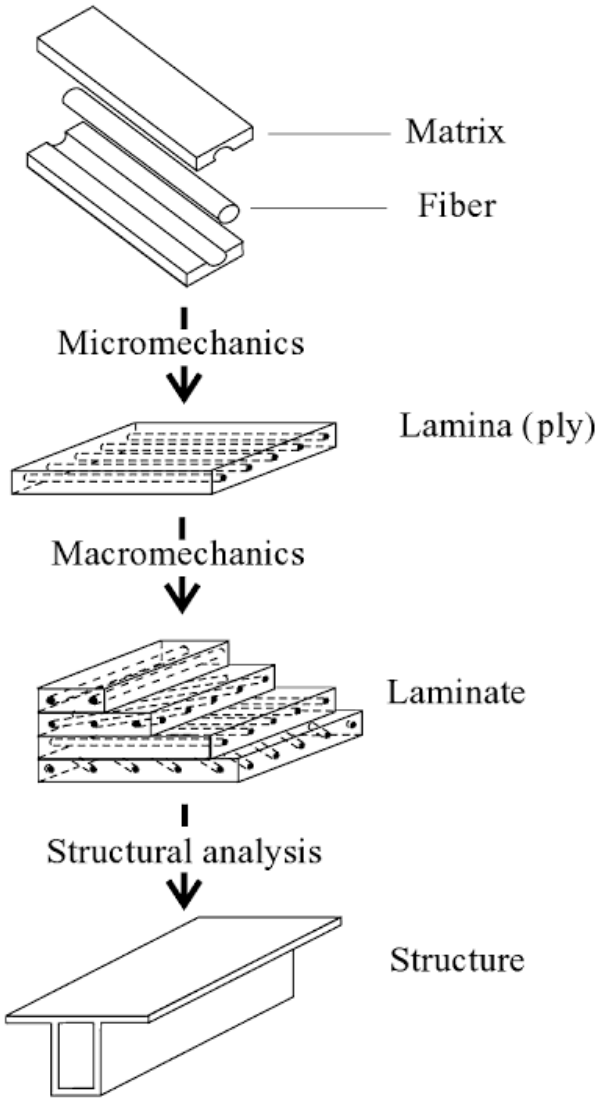
$$\boldsymbol{\varepsilon} = \mathbf{S} \quad \boldsymbol{\sigma}$$

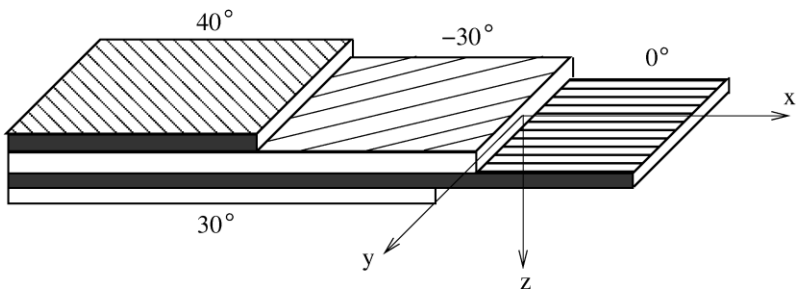
S.... Compliance matrix

Constitutive characteristics

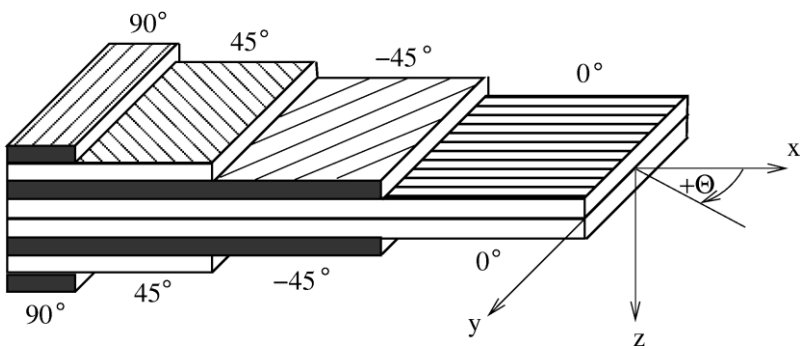
Anisotropic material	no symmetry planes 21 independent material constants
Monoclinic material	1 symmetry plane 13 independent material constants
Orthotropic material	3 symmetry plane 9 independent material constants
Transversally isotropic	3 symmetry plane 5 independent material constants
Isotropic material	3 symmetry plane 2 independent material constants

Thin laminates – laminate code





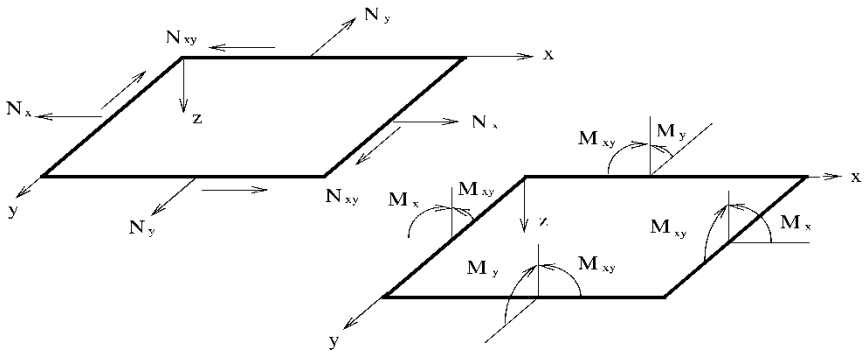
[40/-30/0/30] – nonsymmetric laminate



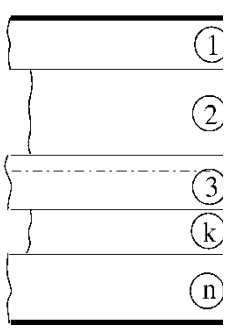
[90/+45/-45/0/0/-45/+45/90]=[90/±45/0]$_s$
– symmetric laminate

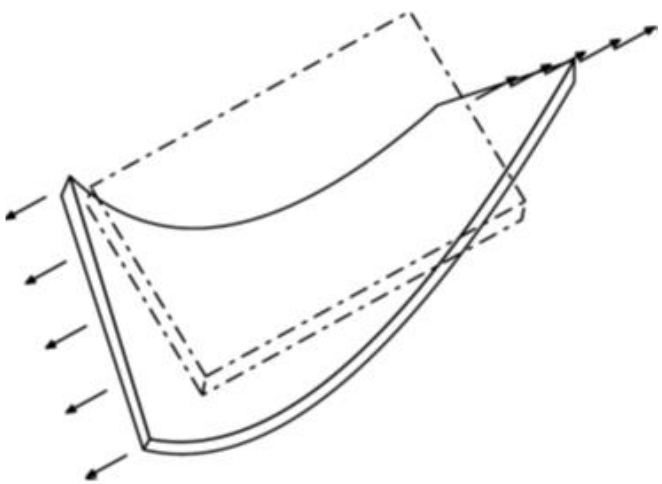
Classical lamination theory

= relation between internal forces N, moments M and strains



$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{21} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\ A_{61} & A_{62} & A_{66} & B_{61} & B_{62} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{21} & B_{22} & B_{26} & D_{21} & D_{22} & D_{26} \\ B_{61} & B_{62} & B_{66} & D_{61} & D_{62} & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon^{\circ}_{xx} \\ \varepsilon^{\circ}_{yy} \\ \gamma^{\circ}_{xy} \\ k_x \\ k_y \\ k_{xy} \end{bmatrix}$$





$$\begin{bmatrix} \mathbf{N} \\ \cdots \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \vdots & \mathbf{B} \\ \cdots & \vdots & \cdots \\ \mathbf{B} & \vdots & \mathbf{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}^{\circ}_{\mathbf{m}} \\ \cdots \\ \mathbf{k} \end{bmatrix}$$

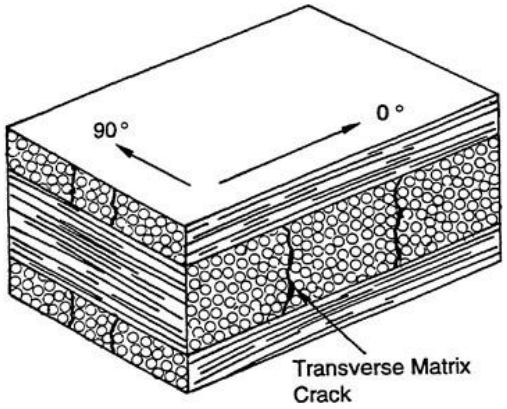
Static failure criteria of laminates

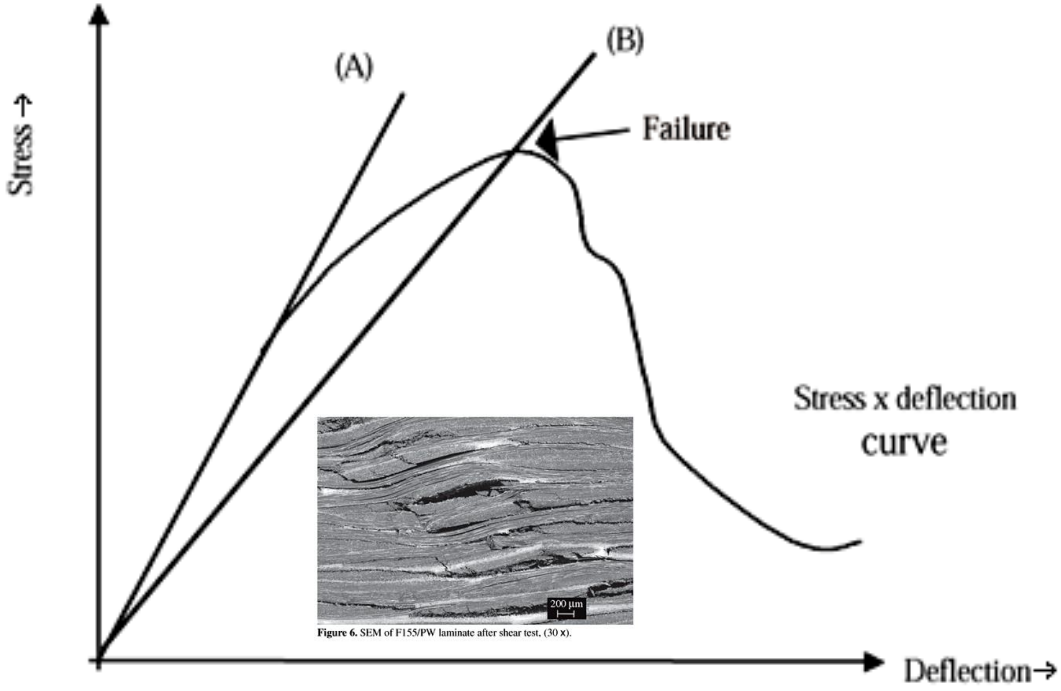
A. Non-interactive criterion

- Maximum stress
- Maximum strain

B. Interactive criterion

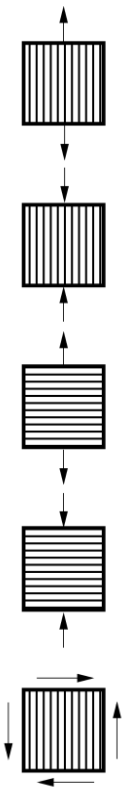
- Tsai-Hill criterion
- Hoffman
- Tsai-Wu criterion
- Puck





J. Braz. Soc. Mech. Sci. vol.22 No.2 Campinas 2000

Static failure criteria of laminates



F_{Lt} Longitudinal tensile strength

F_{Lc} Longitudinal compression strength

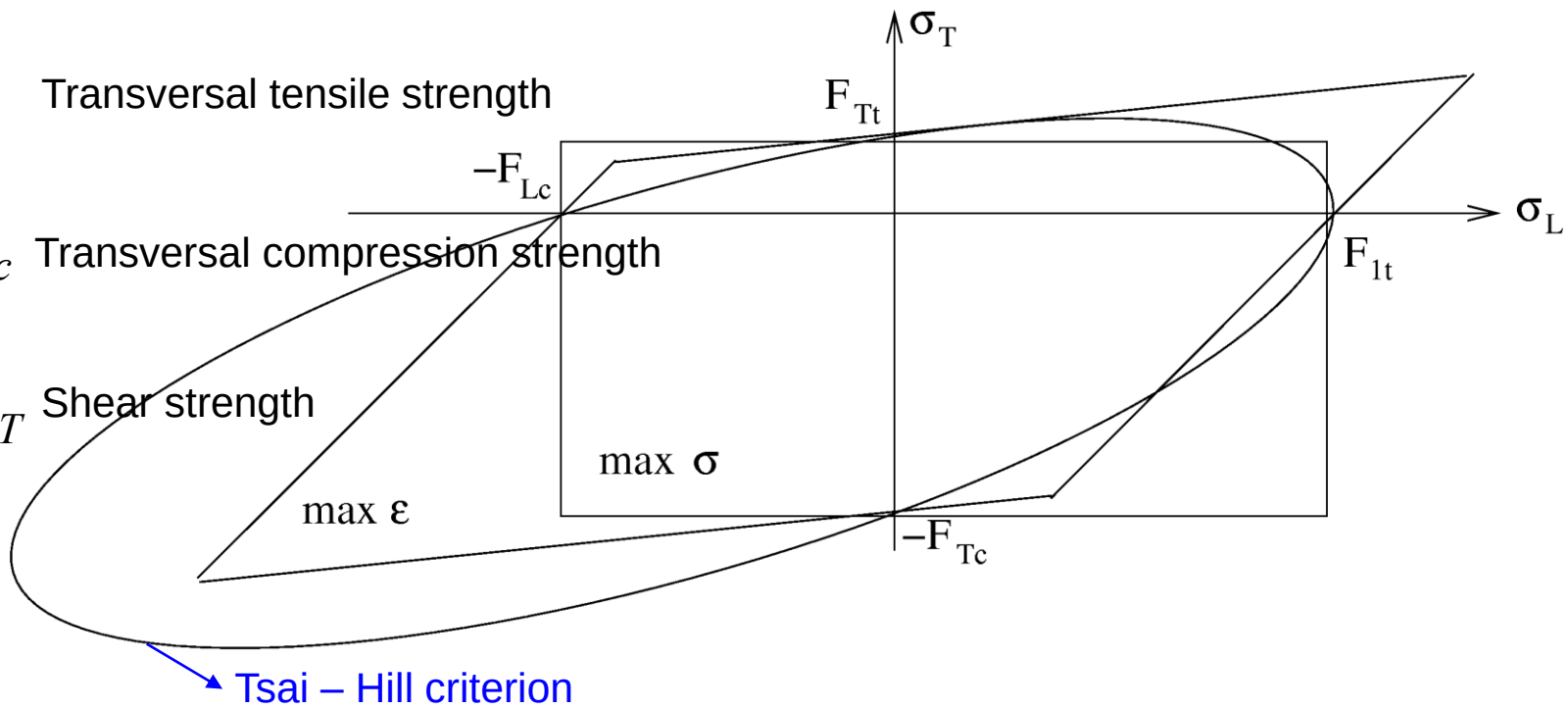
F_{Tt} Transversal tensile strength

F_{Tc} Transversal compression strength

F_{LT} Shear strength

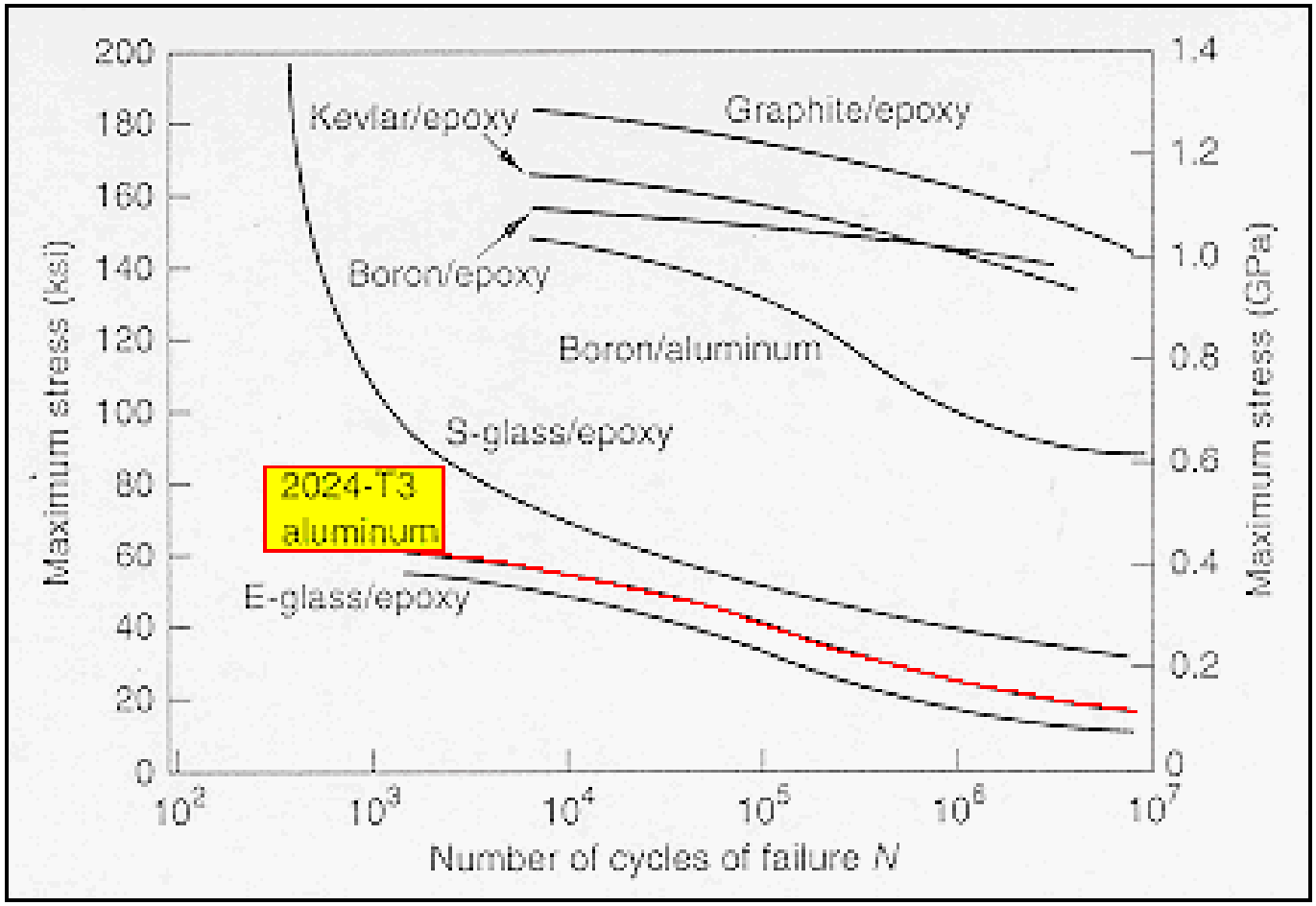
Tsai – Hill criterion

$$\left(\frac{\sigma_L}{F_L}\right)^2 + \left(\frac{\sigma_T}{F_T}\right)^2 + \left(\frac{\sigma_{LT}}{F_{LT}}\right)^2 - \frac{\sigma_L \sigma_T}{F_L^2} = 1$$



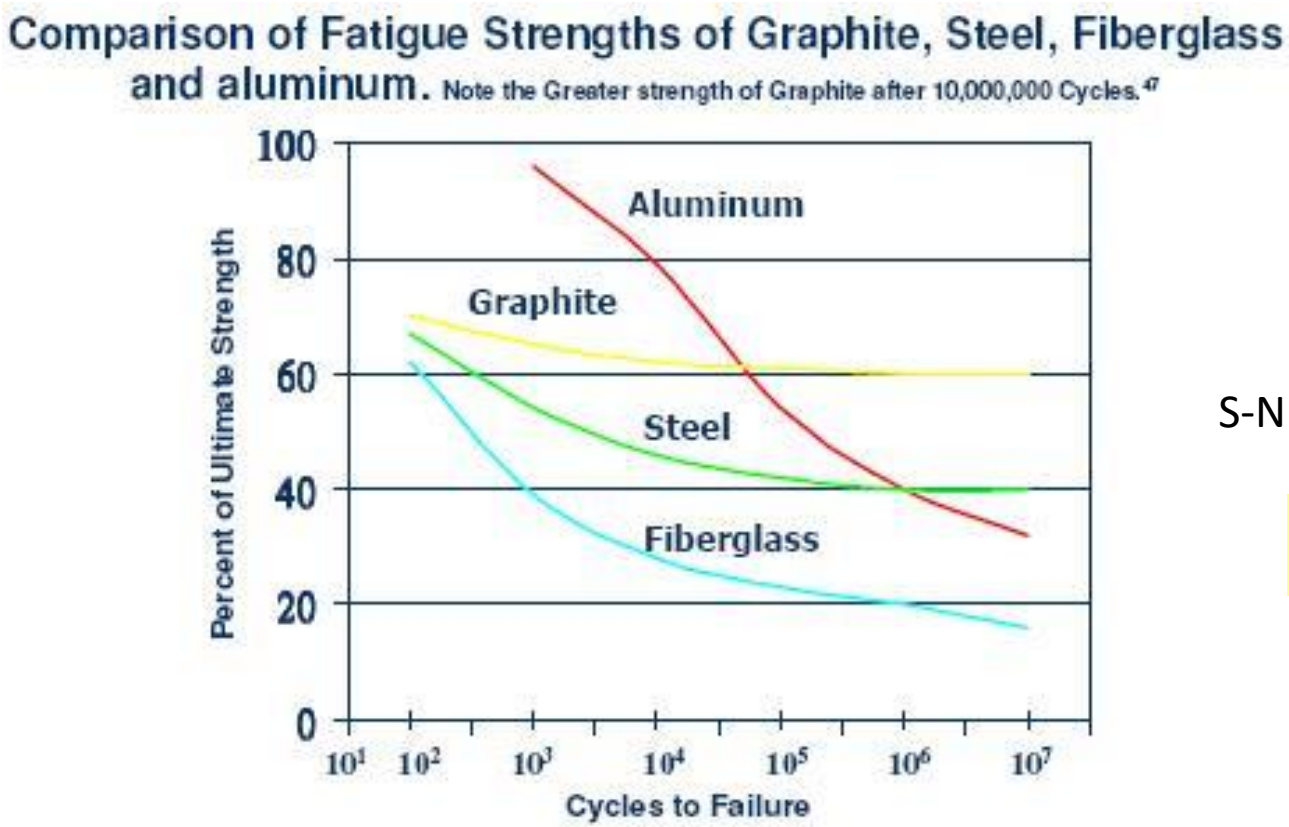
Fatigue properties of laminates

Stress-life curves comparison (R=0.1)



W.F.Smith: "Principles of Materials Science & Engineering," McGraw Hill (1986)

Fatigue properties of laminates



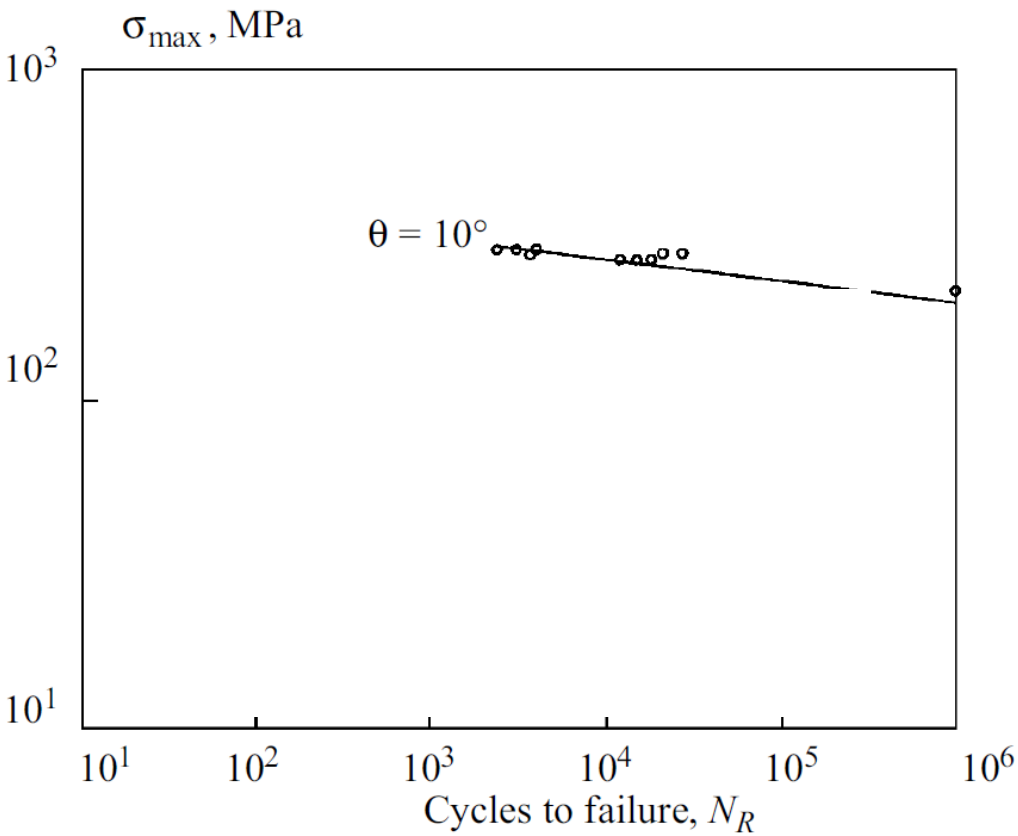
http://www.compositesblog.com/2010/03/composites-vs-metal.html

S-N curve description

$$\sigma_a^m N = C$$

Fatigue curves for unidirectional laminates

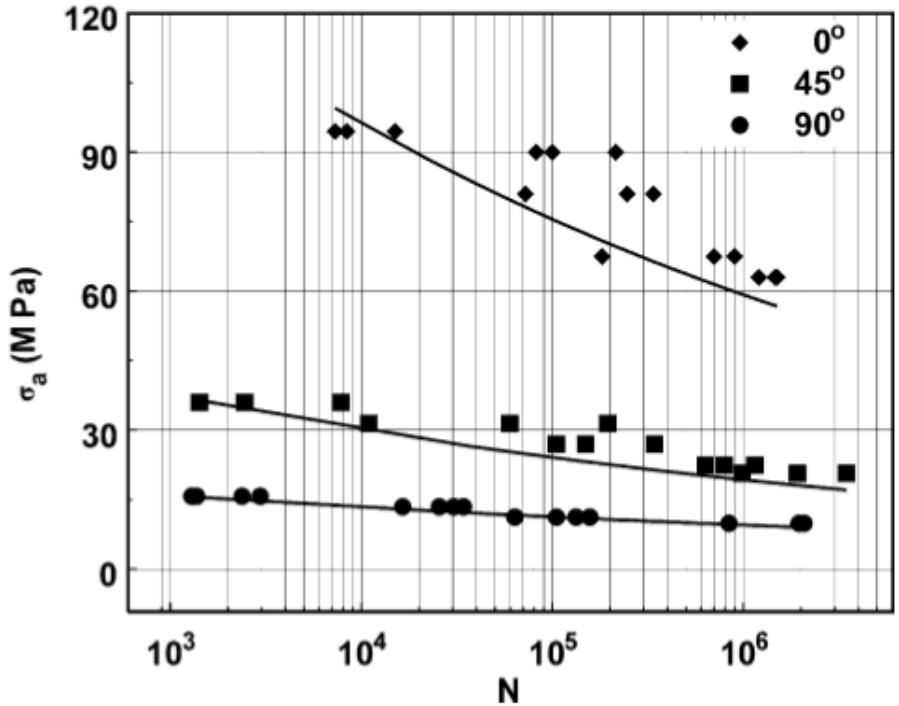
Carbon Fibre Reinforced Plastics



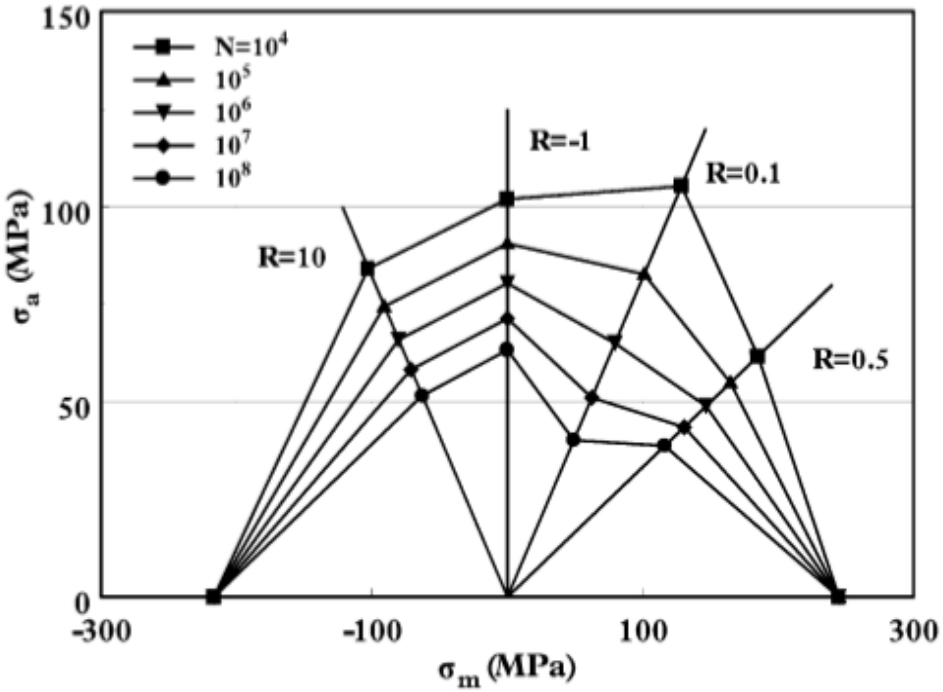
D. Revuelta and A. Miravete: *International Applied Mechanics, Vol. 38, No. 2, 2002*

Fatigue curves for unidirectional laminates

Glass Fibre Reinforced Plastics



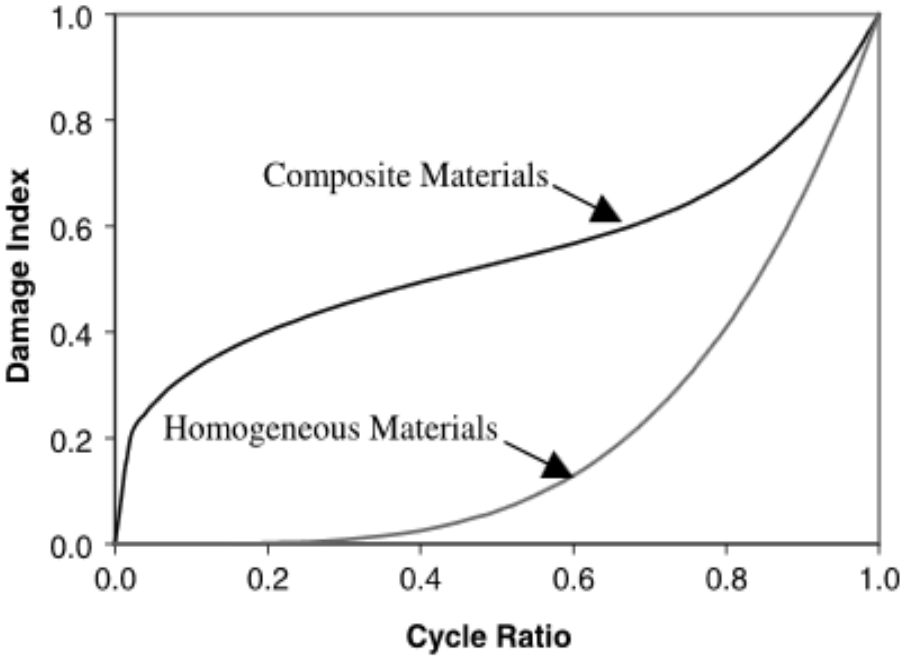
S–N curves at 95% reliability level. $R = 0.1$, $\theta = 0°$, 45° and 90°.

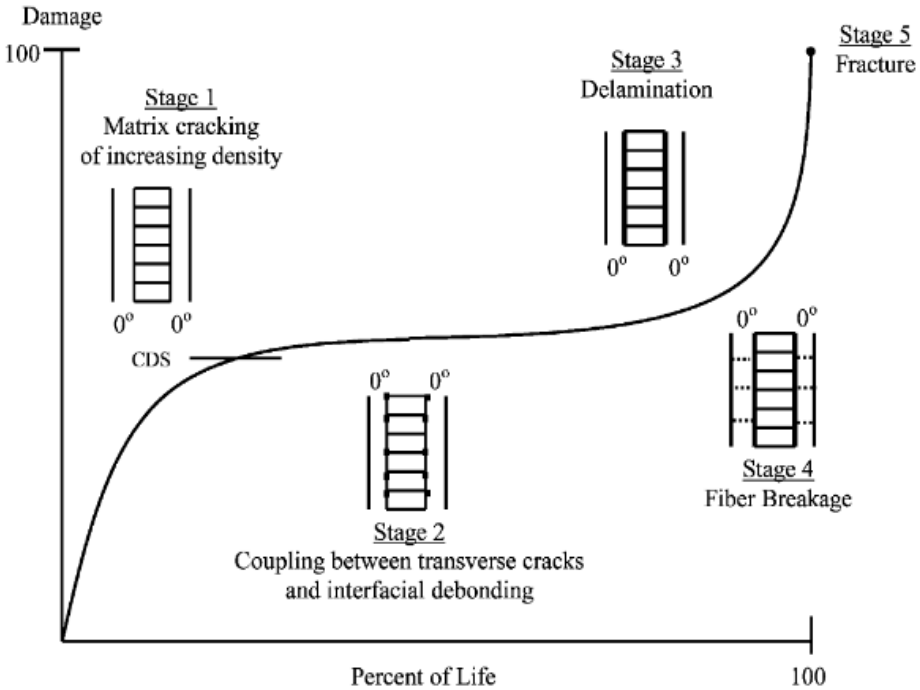


Constant life diagram for on-axis, 0°, coupons.

T.P. Philippidis, A.P. Vassilopoulos / International Journal of Fatigue 24 (2002) 813 –823

Fatigue damage description





H. Mao, S. Mahadevan / Composite Structures 58 (2002) 405–410

Talreja R. Fatigue of composite materials. Lancaster, PA: Technomic Publishing Company, Inc; 1987. p. 20.

$$\frac{dD}{dN} = f(\sigma, R, D \dots)$$ …damage cummulation velocity

Fatigue damage description

Fatigue description:

-Phenomenological methods

-Deterministic methods

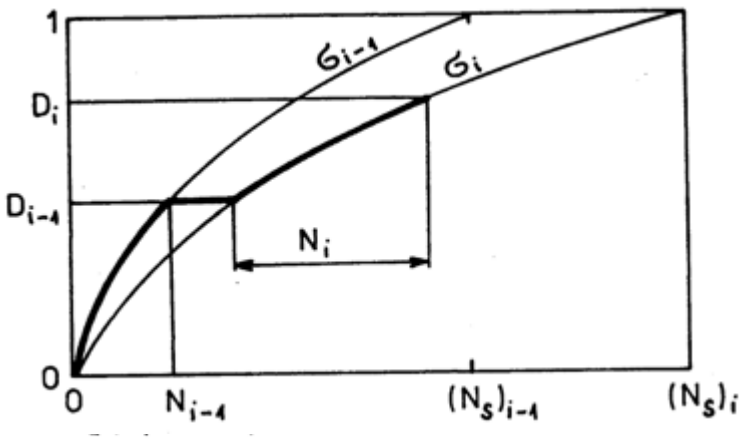
-Statistical methods

Fatigue damage $D = f(\sigma_m, \sigma_a, f, n_i)$

The most frequently model is linear:

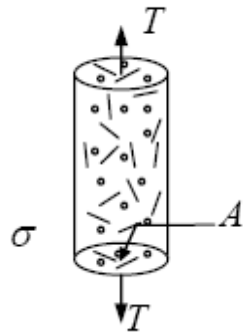
Miner's rule $D = \Sigma (n_i / N_{fi})$
gives failure for $D \approx 1$
but often $D \neq 1$!!!

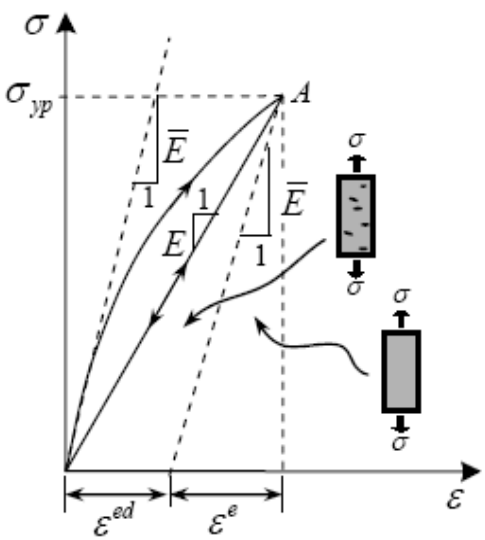
D cannot not be valid exactly since there are fatigue life periods



Continuum damage model

Real specimen
Configuration with damage





(a)

Equivalence of tensile forces:

$$T = \sigma.A = \bar{\sigma}.\bar{A}$$

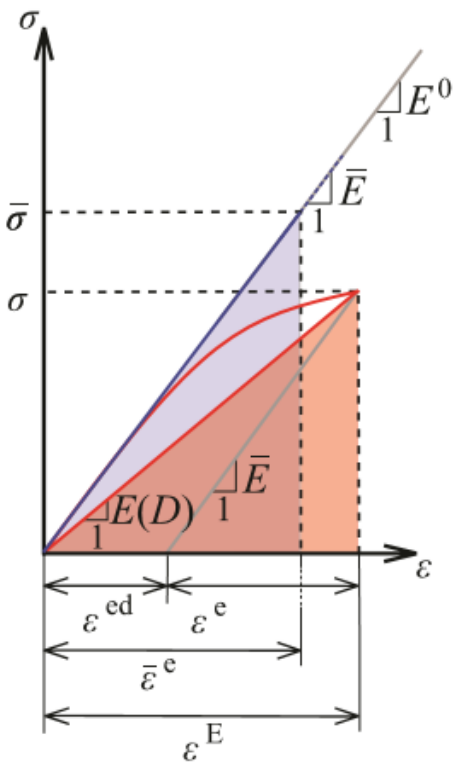
Damage parameter *D*

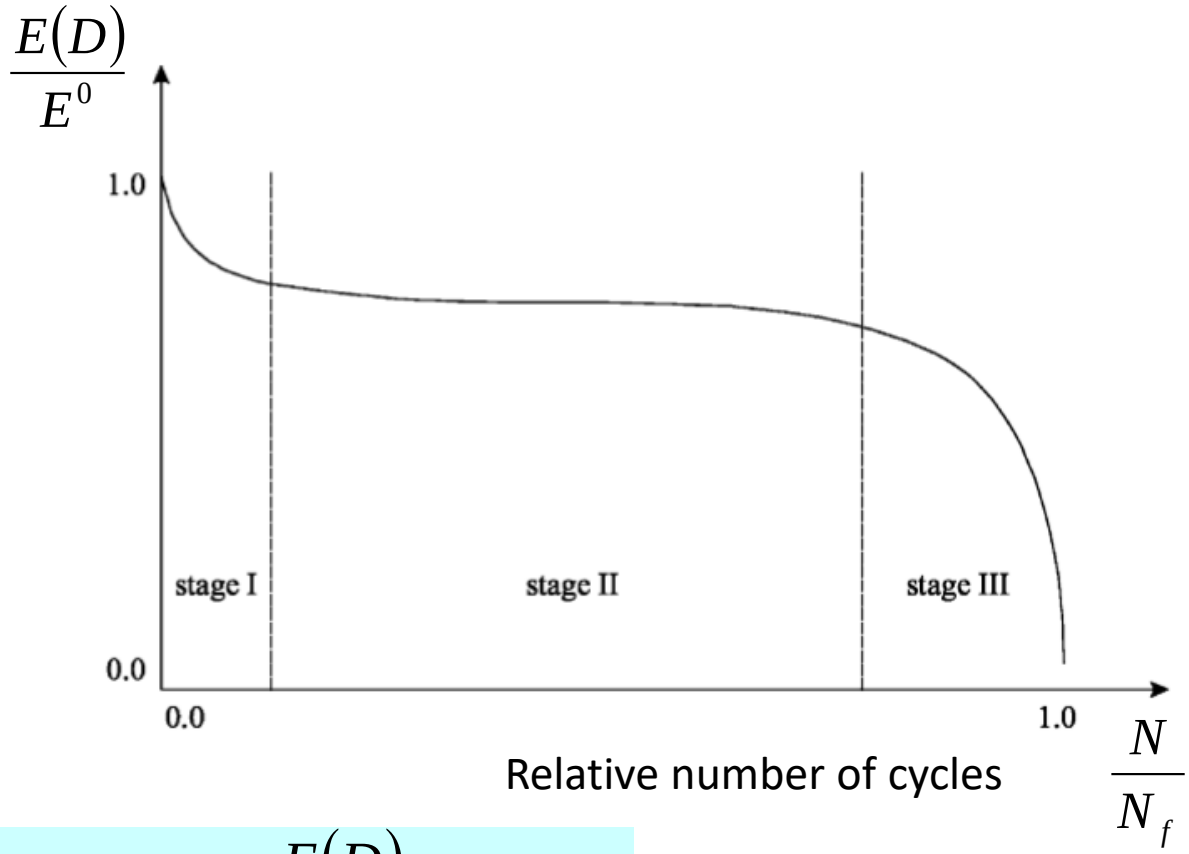
$$D = \frac{A - \bar{A}}{A} = \frac{A^D}{A}$$

True and equivalent stress relationship:

$$\bar{\sigma} = \frac{\sigma}{1 - D}$$

E-modulus degradation



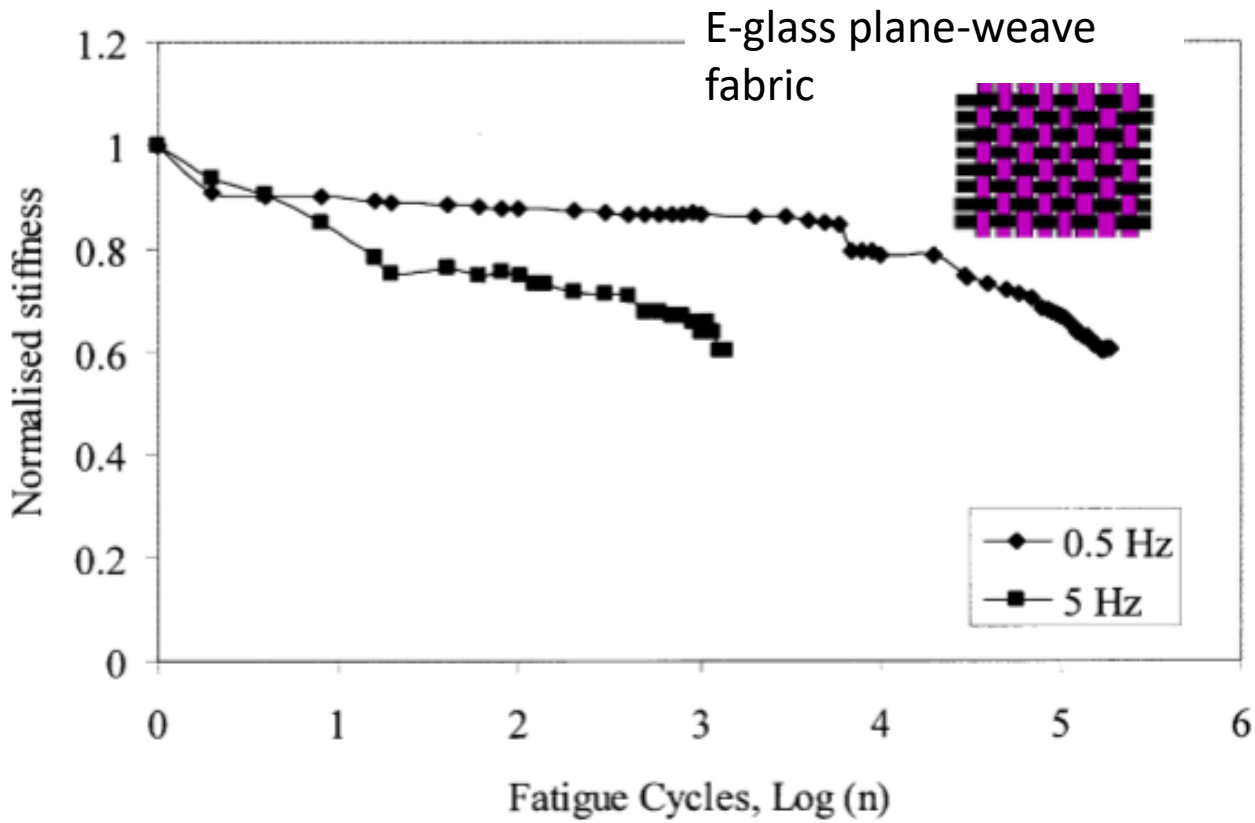


$$\sigma = E^0 \cdot (1-D) \cdot \varepsilon^E = E(D) \cdot \varepsilon^E, \qquad \frac{E(D)}{E^0} = (1-D)$$

$$D = 1 - \frac{E(D)}{E^0}$$

E-modulus degradation

$$D = 1 - \frac{E(D)}{E^0}$$



Pandita at. al: Composites, Part A 32 (2001) 1533-1539

Progress of the damage

$\frac{dD}{dN} = f(\Delta\sigma, R, D \ldots)$ ...damage cummulation velocity

$$\frac{dD}{dN} = \left[\frac{\sigma_a}{M(\sigma_m).(1-D)}\right]^{2q}$$

Lemaitre,J-Desmorat,R.:
Engineering damage mechanics.
Springer 2005

Influence of the mean stress

$$M(\sigma_m) = \left(\frac{q}{2C}\right)^{\frac{1}{2q}}\left(1 - m\frac{\sigma_m}{\sigma_f}\right)$$

where, parameters of the S-N curve are:

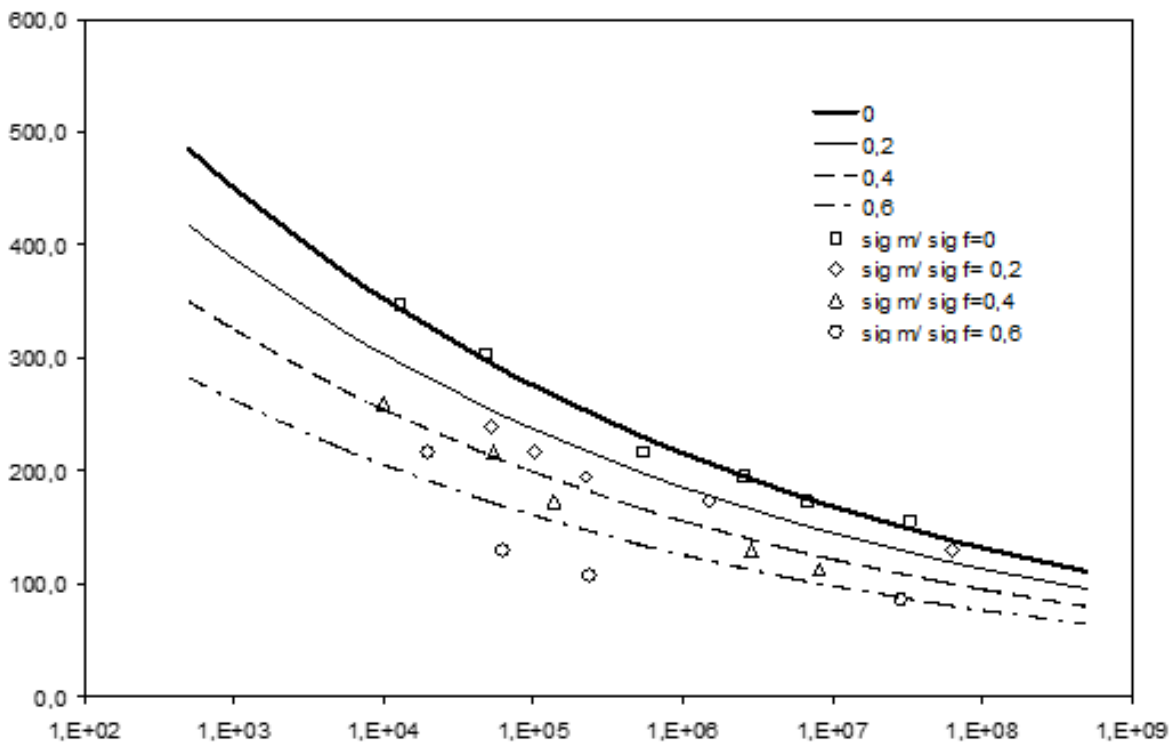
$$\sigma_a^m N = C$$

Fatigue life evaluation

$$\frac{dD}{dN} = f(\Delta\sigma, R, D \ldots)$$ ...damage cumulation velocity

$$N_f = \int_{D_i}^{D_f} \frac{dD}{f(\Delta\sigma, R, D)}$$

$$N_f = \frac{q.\left(1 - m.\frac{\sigma_m}{\sigma_f}\right)^{2q}}{2.(2q+1).C}.\sigma_a^{-2q} = X.\sigma_a^{-2q}$$



Thank you for your attention